WHY DO WE NEED MODERN GEOMETRY?

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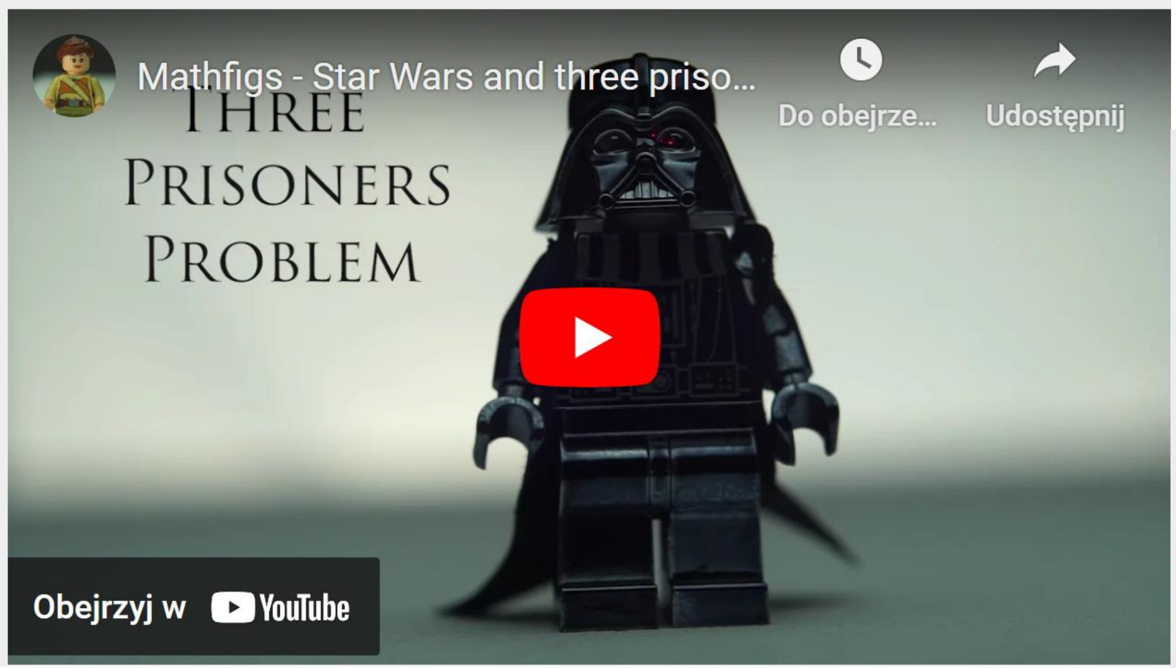
Faculty of Mathematics and Information Science, Warsaw University of Technology

Polish-Japanese Singularity Working Days 2024

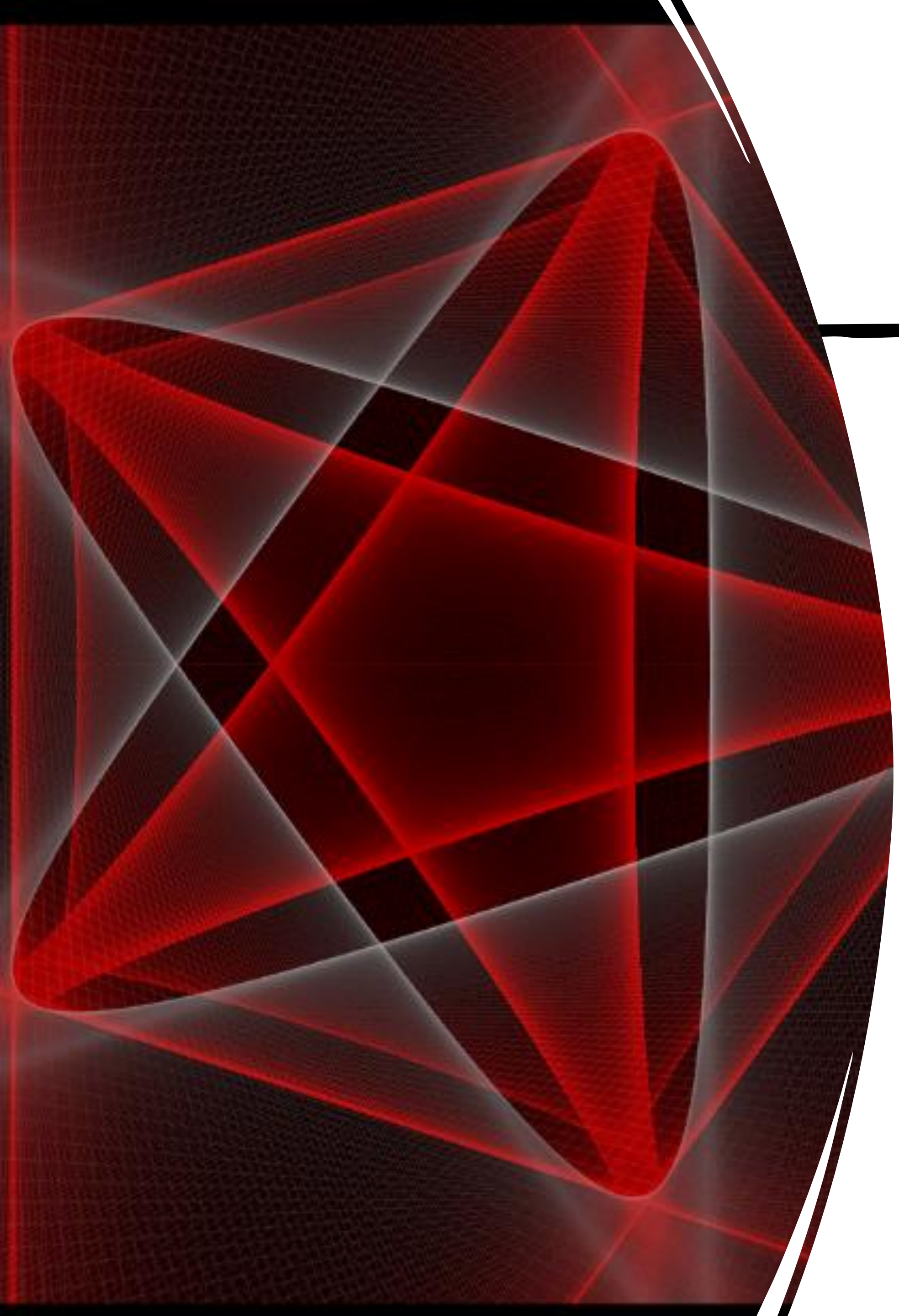
09-14 September 2024

BUT FIRST… ADVERTISEMENT

- YouTube Channel: MathFigs



MY SHORT STORY
RELATED TO P-J CONFERENCES
AND OUR DEPARTMENT



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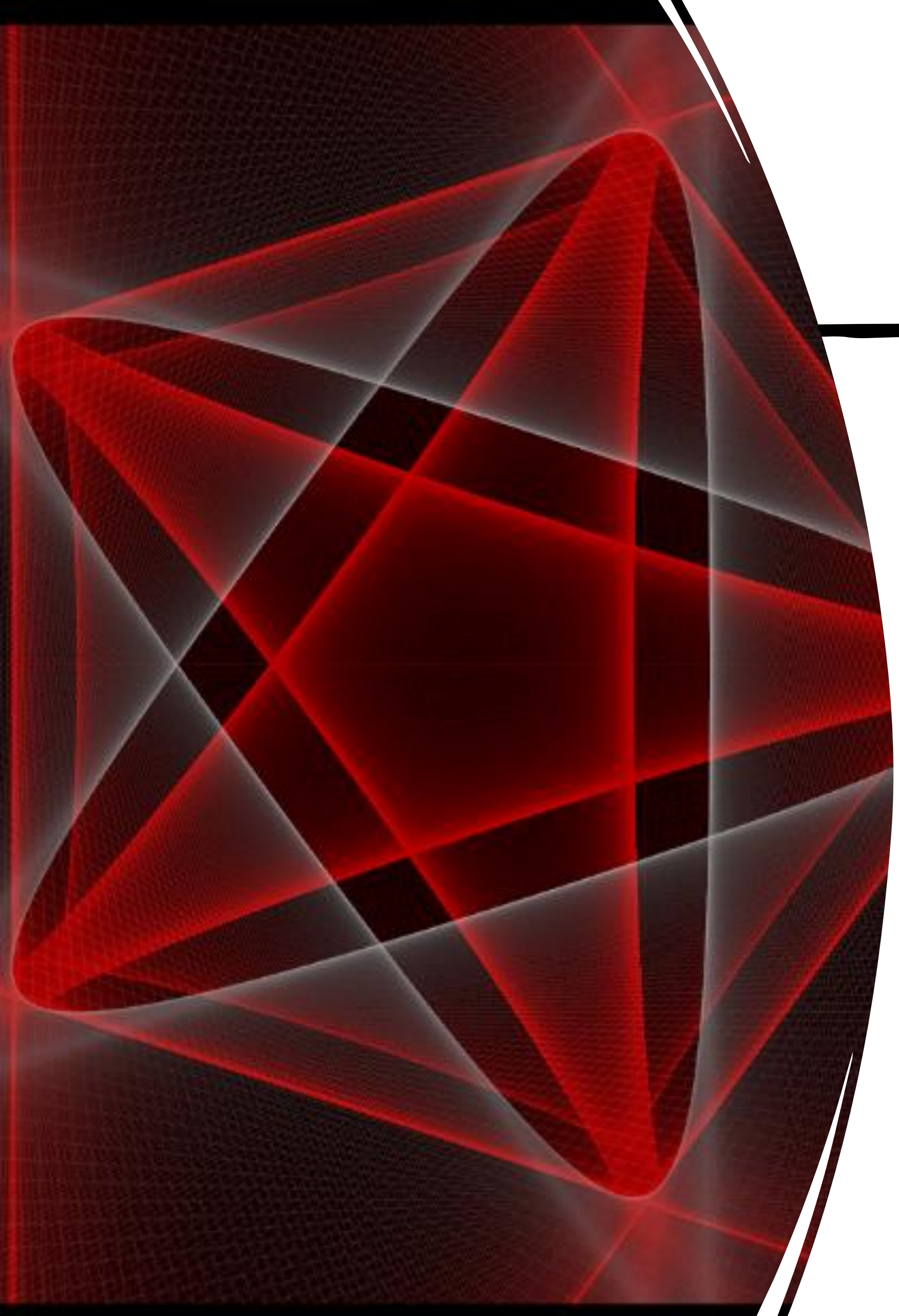

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- My 1st conference was *Geometric Singularity Theory, Polish-Japanese Singularity Working Days, Banach Center Conferences, Warsaw, 24-31.08.2013.*

FIRST, LET'S REFER TO JAPAN ☺

折り紙

GPT 4o

折り紙

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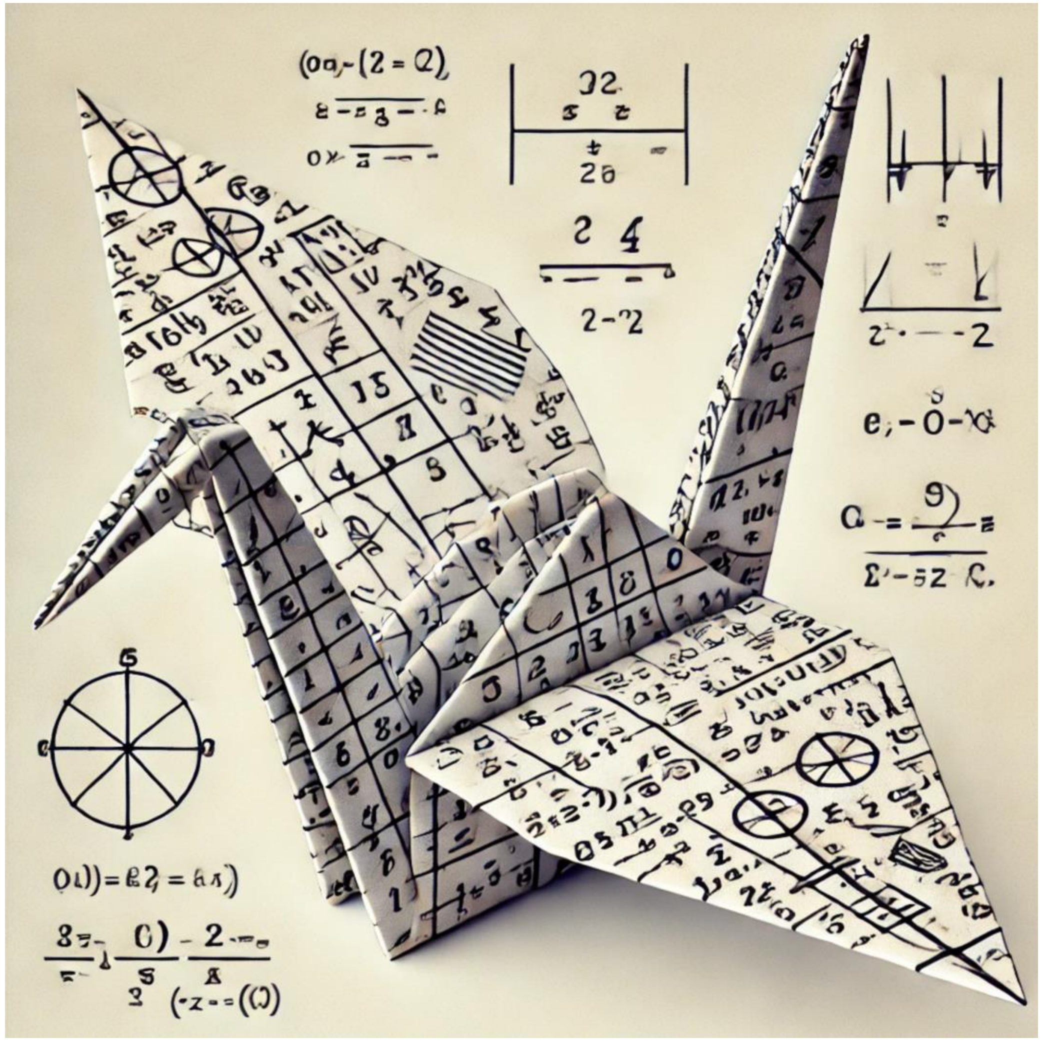
- Origami – art of paper folding



GPT 4o

HUGO STEINHAUS

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„A Mathematician Will Do It Better!"

IS IT CAKE?

IS IT ORIGAMI?


TOP
10

ROBERT J. LANG

- American physicist

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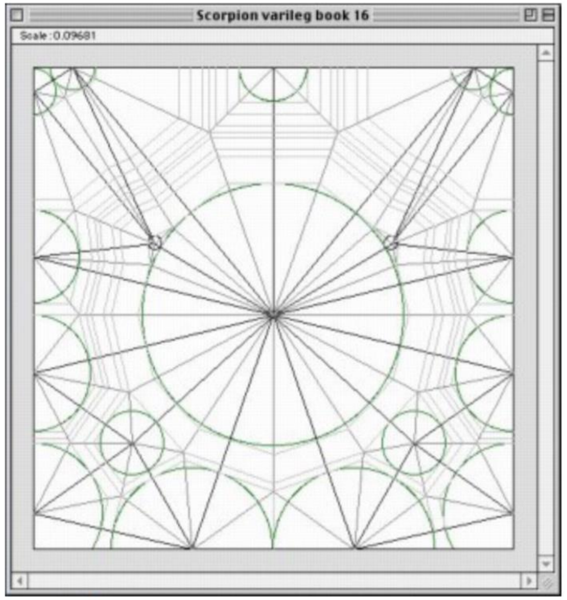
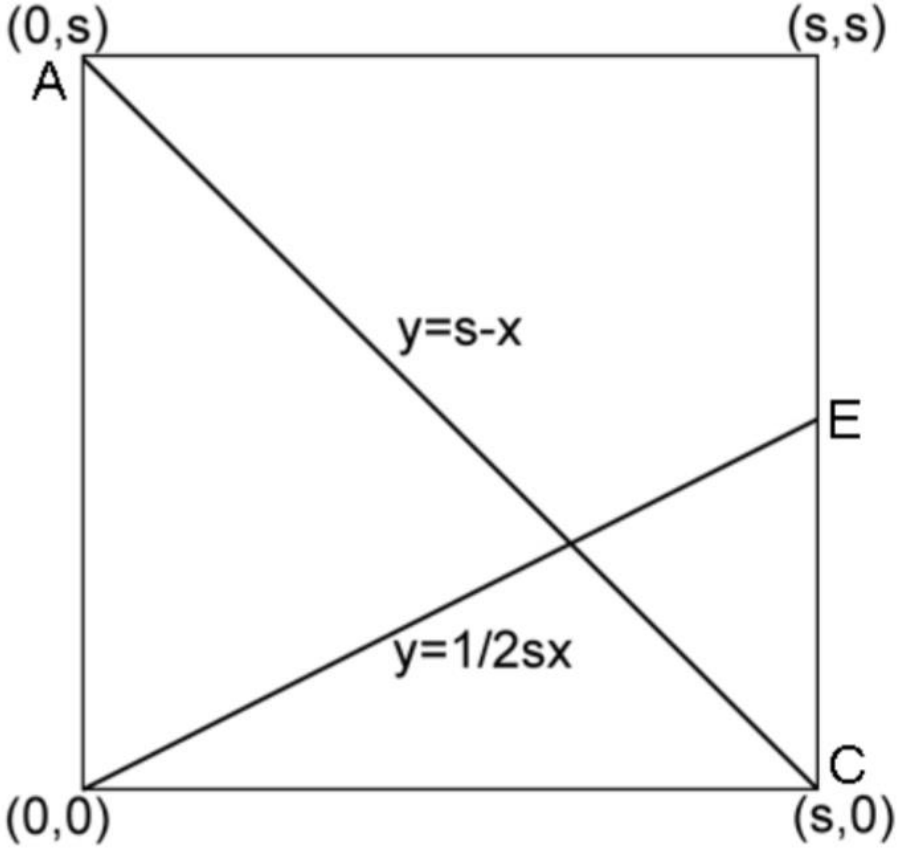
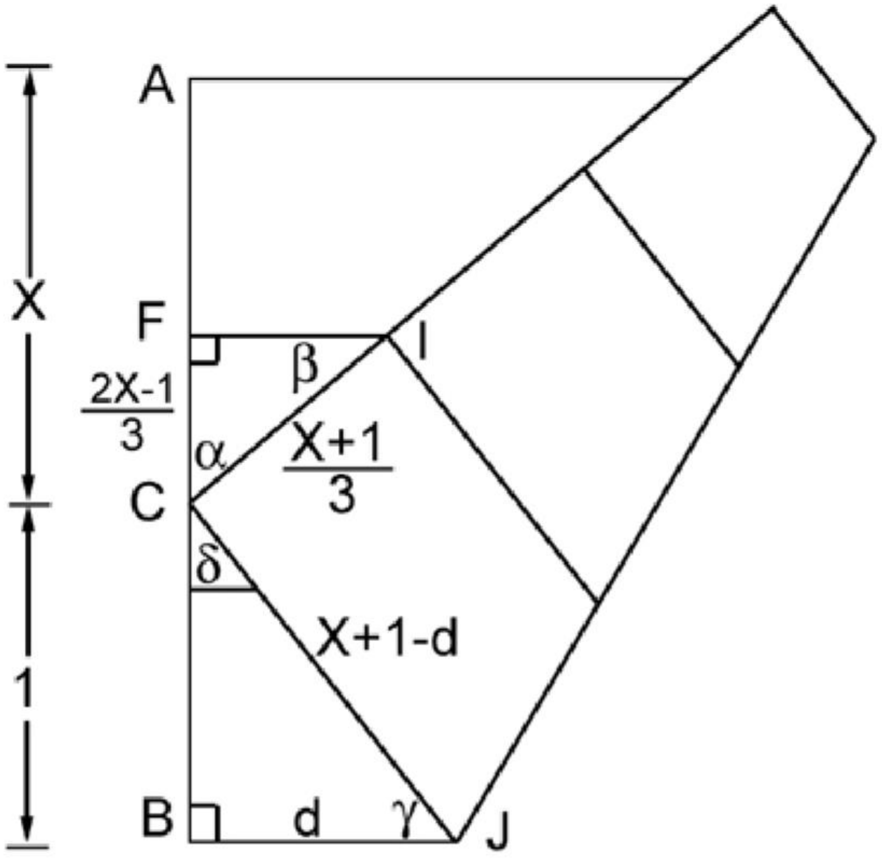


FOLDING PAPER CAN DOUBLE THE CUBE!





$$X = \sqrt[3]{2}$$

SOME MATHEMATICS

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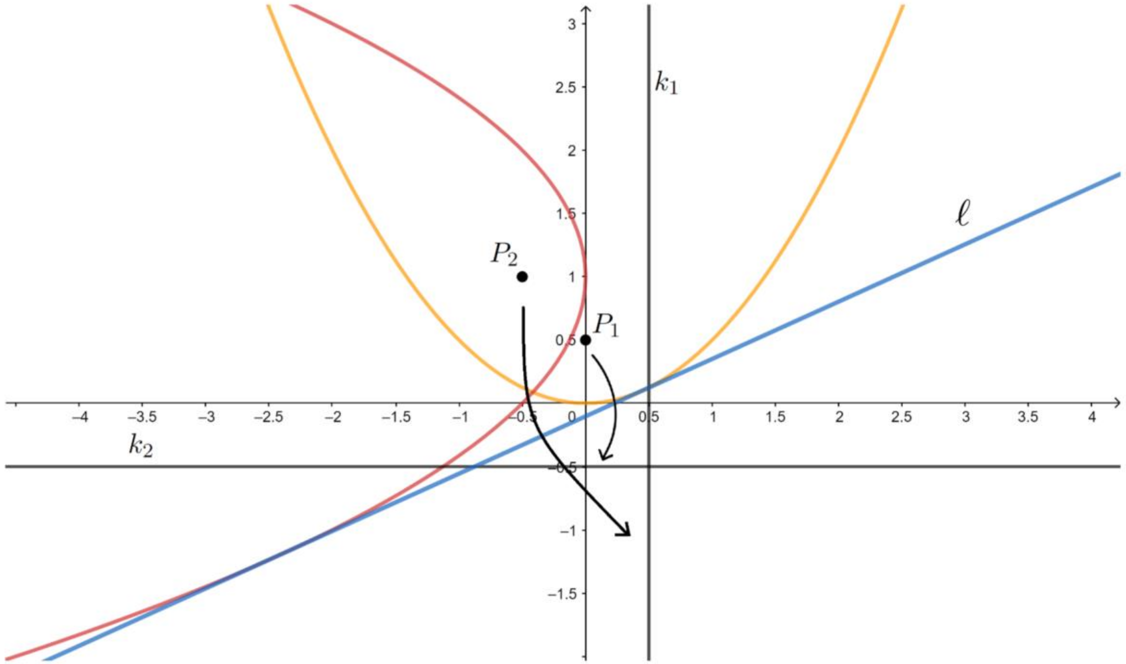
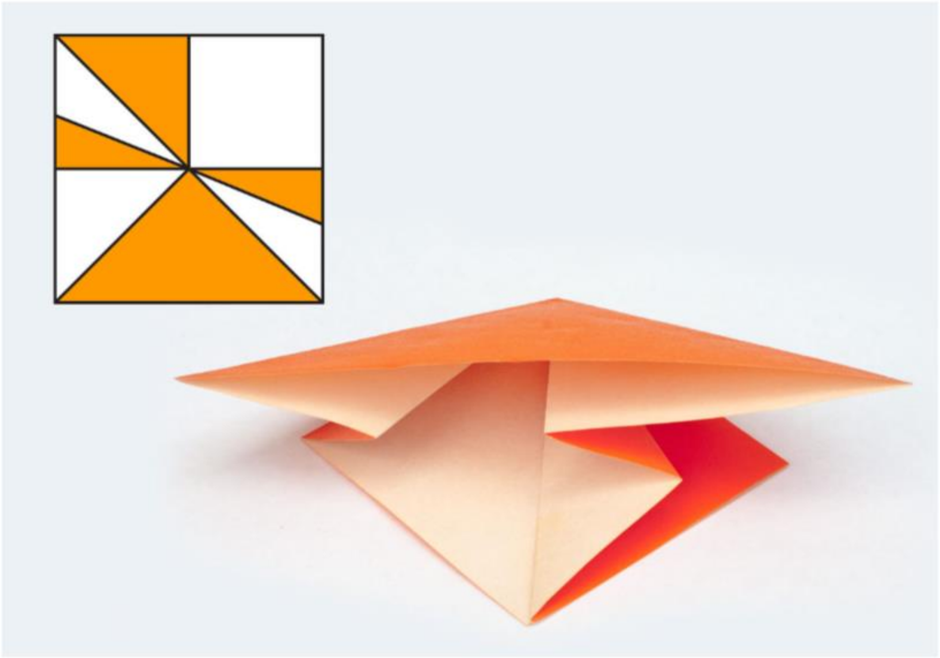
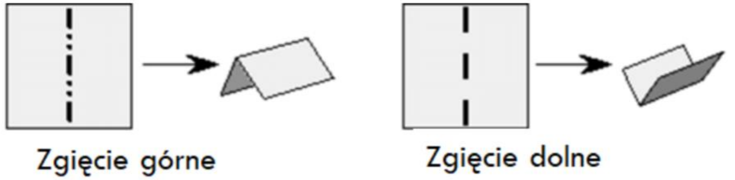
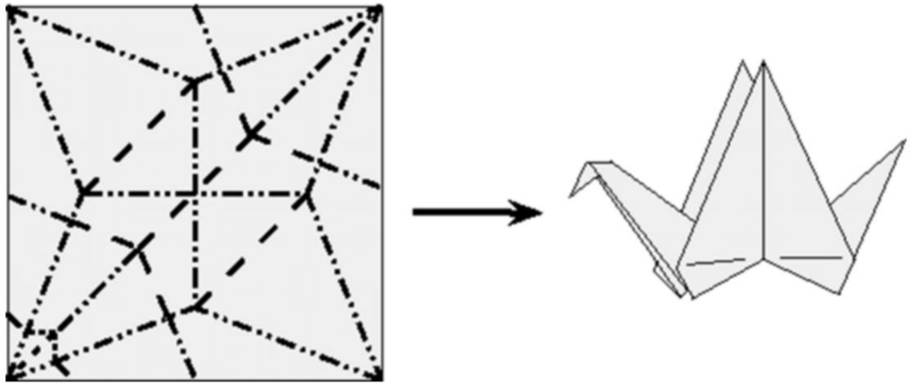


FLAT ORIGAMI

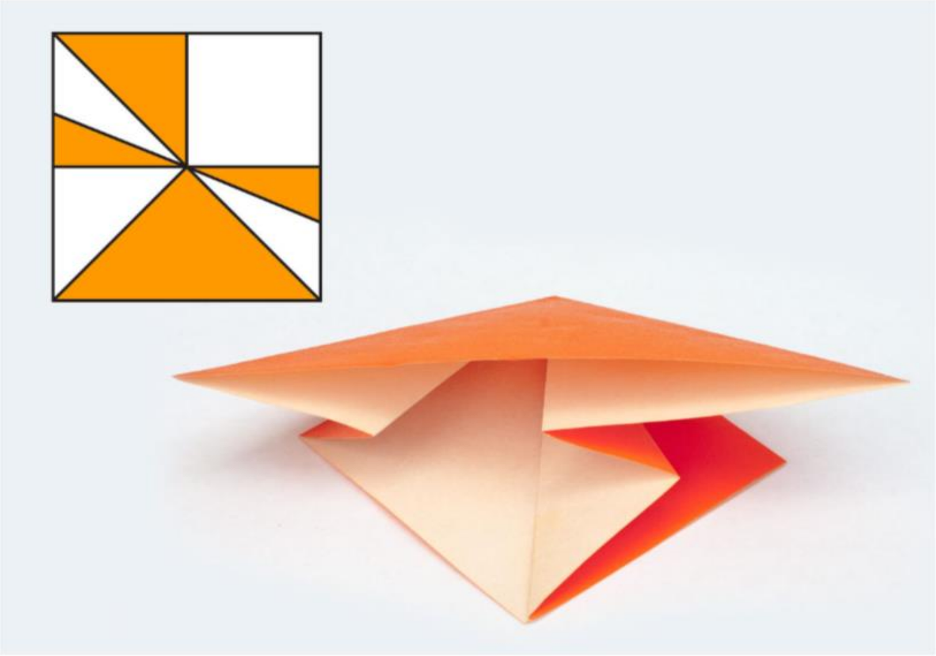


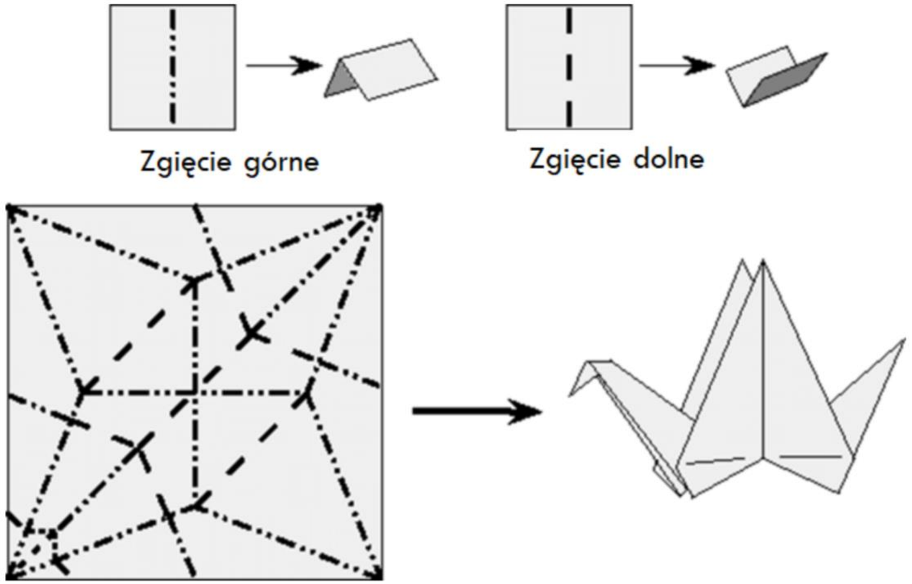




FLAT ORIGAMI

- Thm (Maekawa). *Let U/ D be the number of up/ down creases of a flat origami. Then* $|U-D|=2$.





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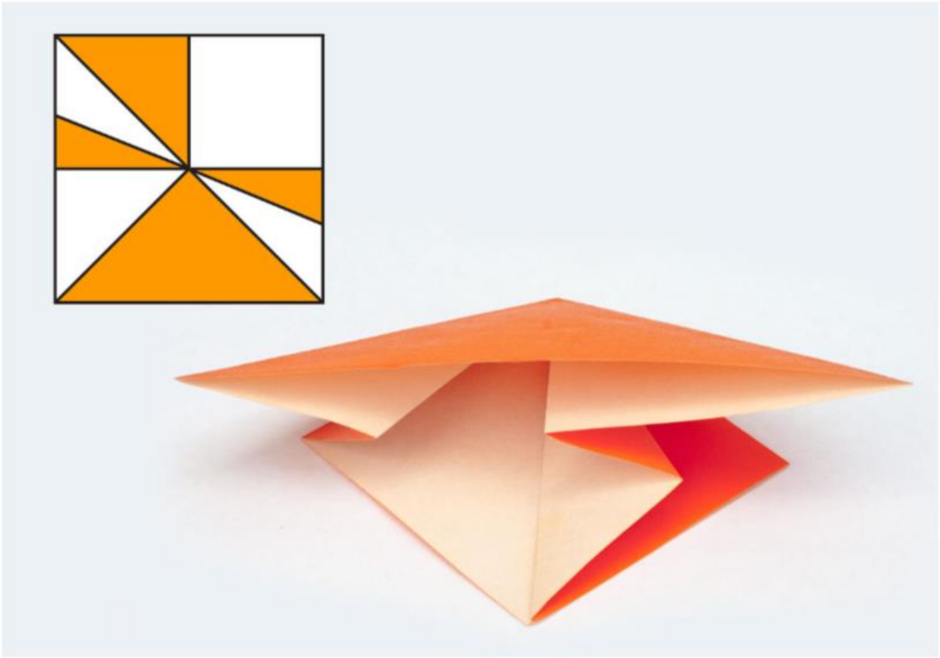

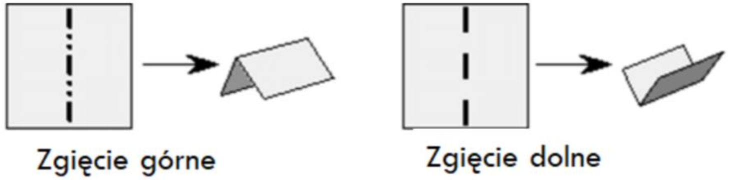

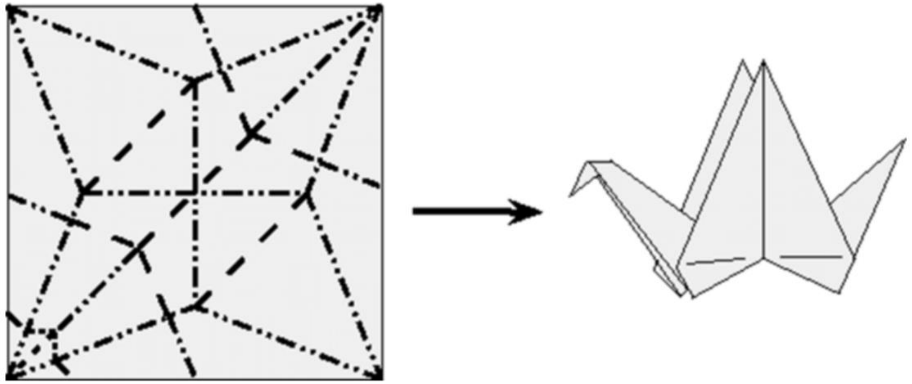

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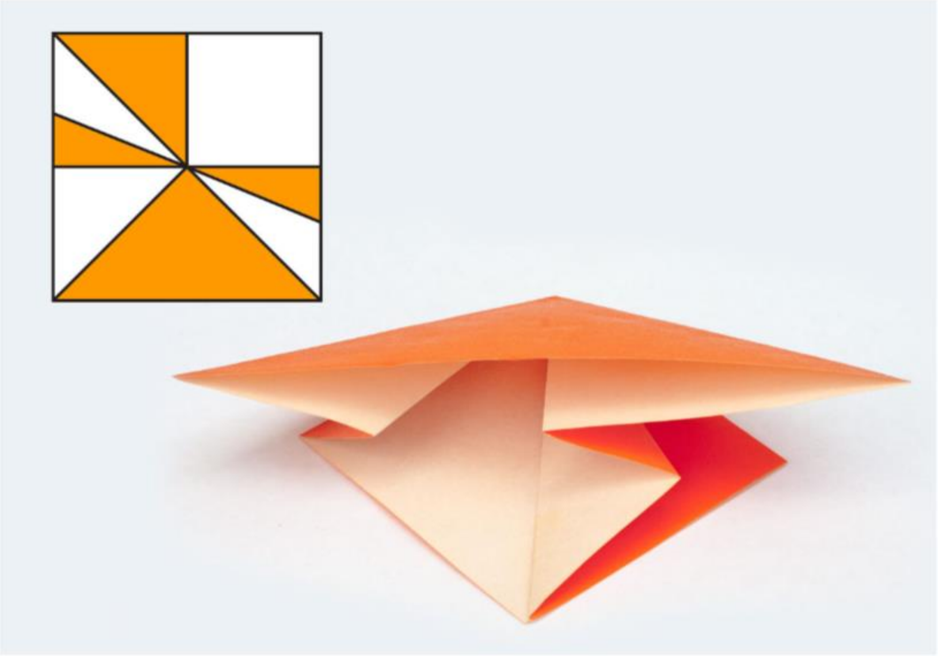

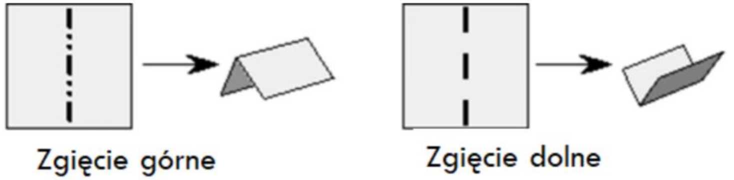

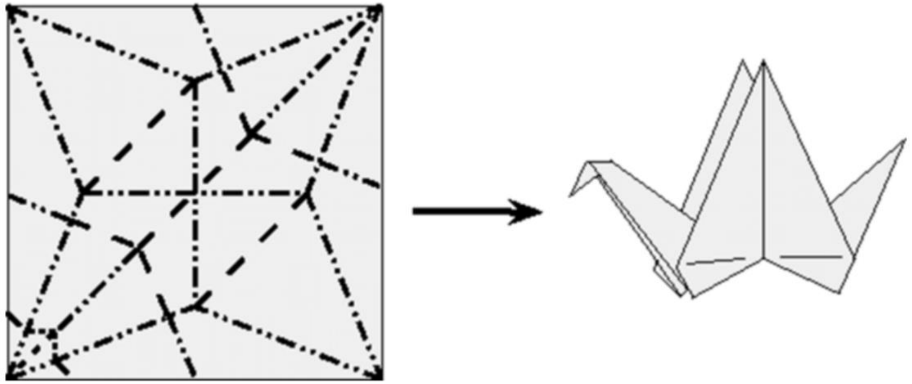

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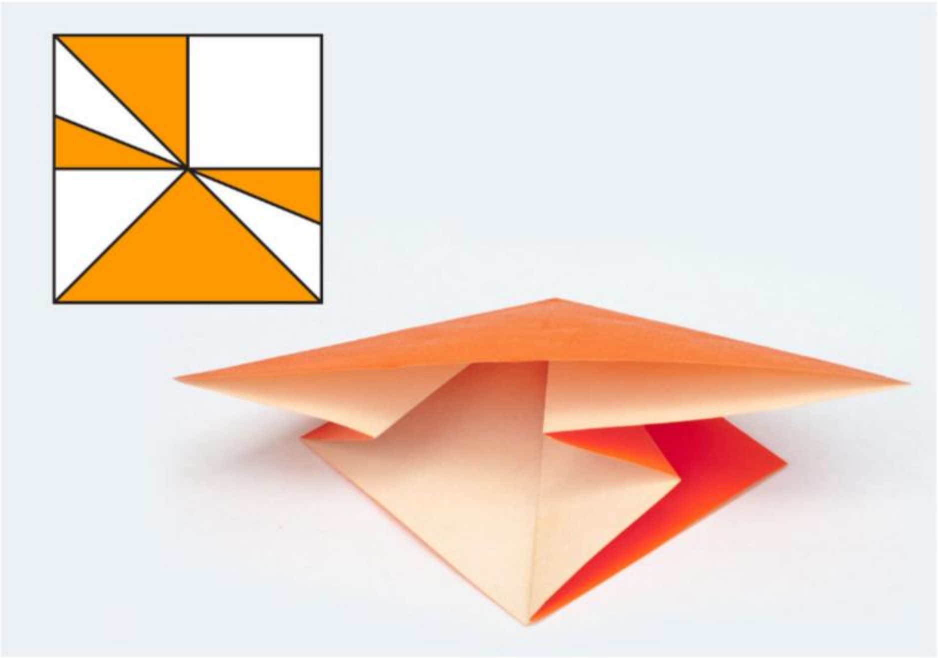
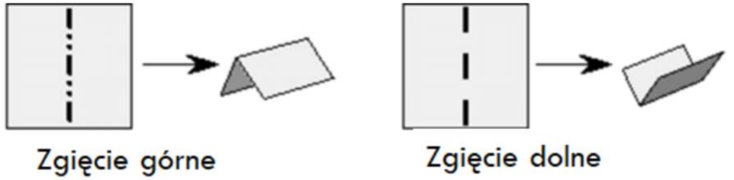
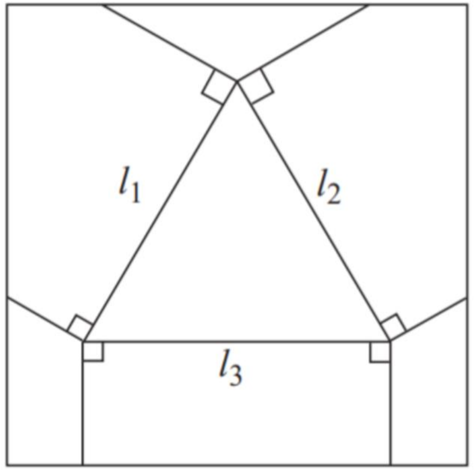
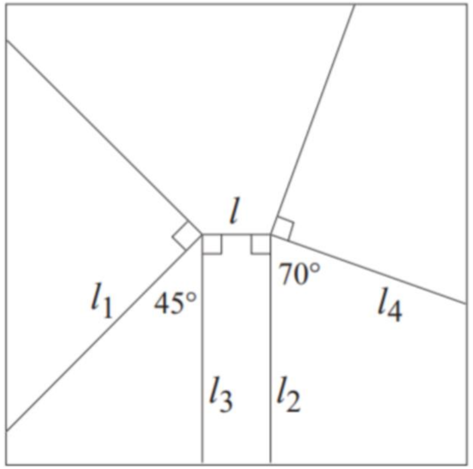
- NP.–hard problem

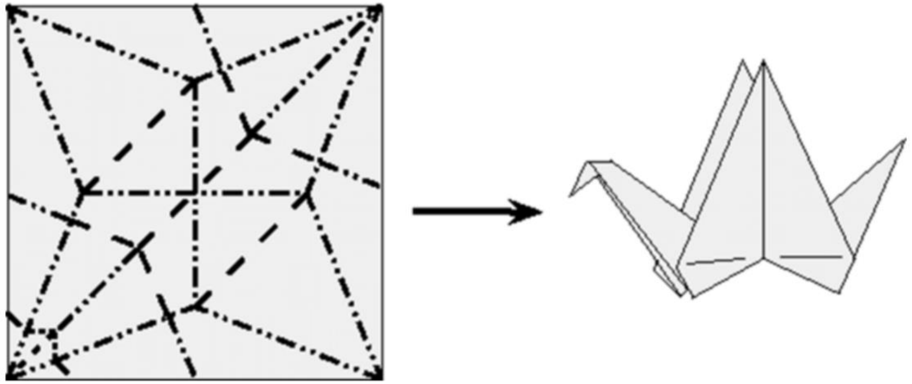
(Bern, Hayes, 1996)





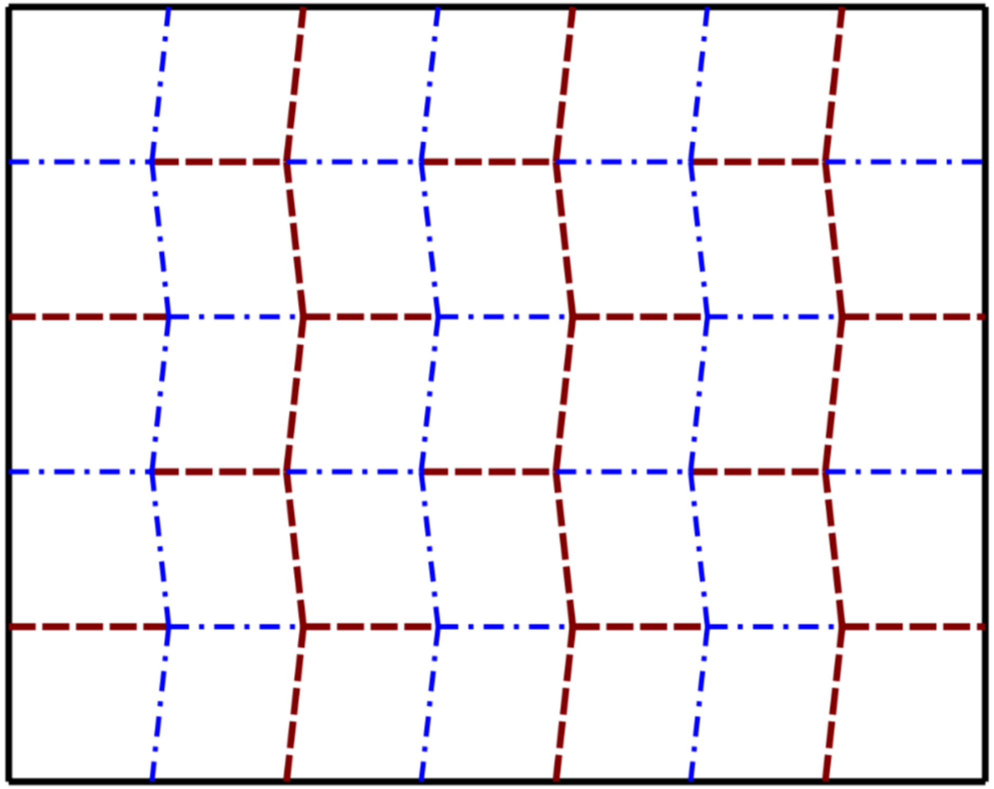






MIURA'S FOLD

- Japanese astrophysicist Koryo Miura in 1970 discovered a fold pattern that allows surfaces made of rigid materials to be folded, and also unfolds very easily



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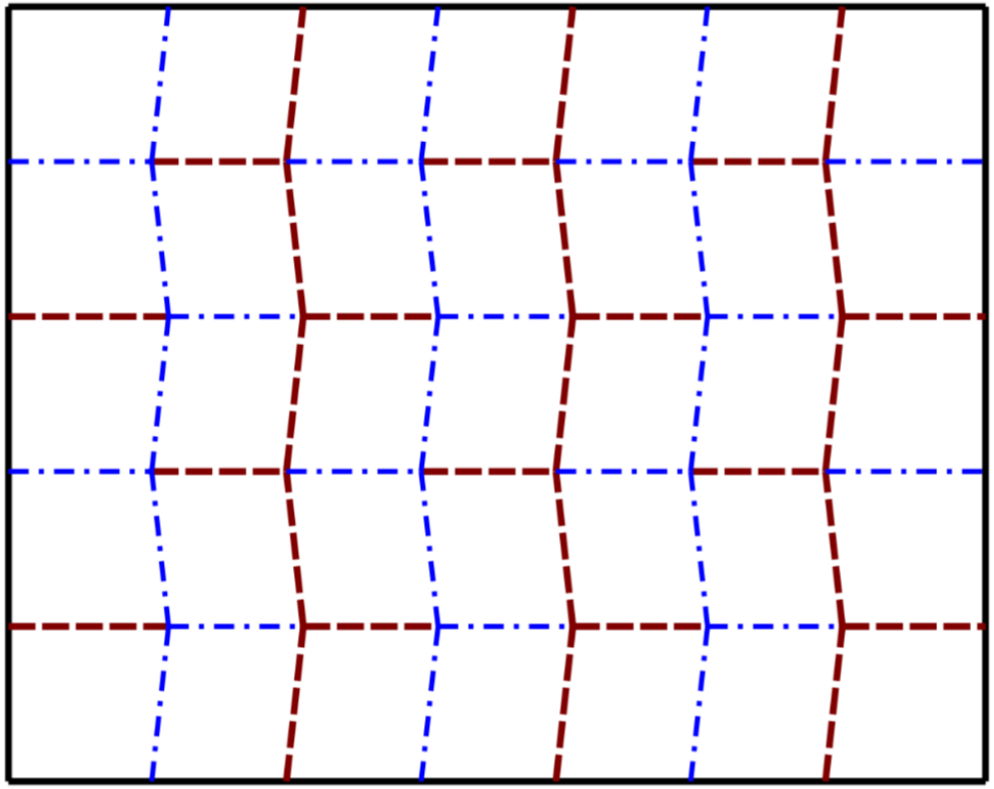

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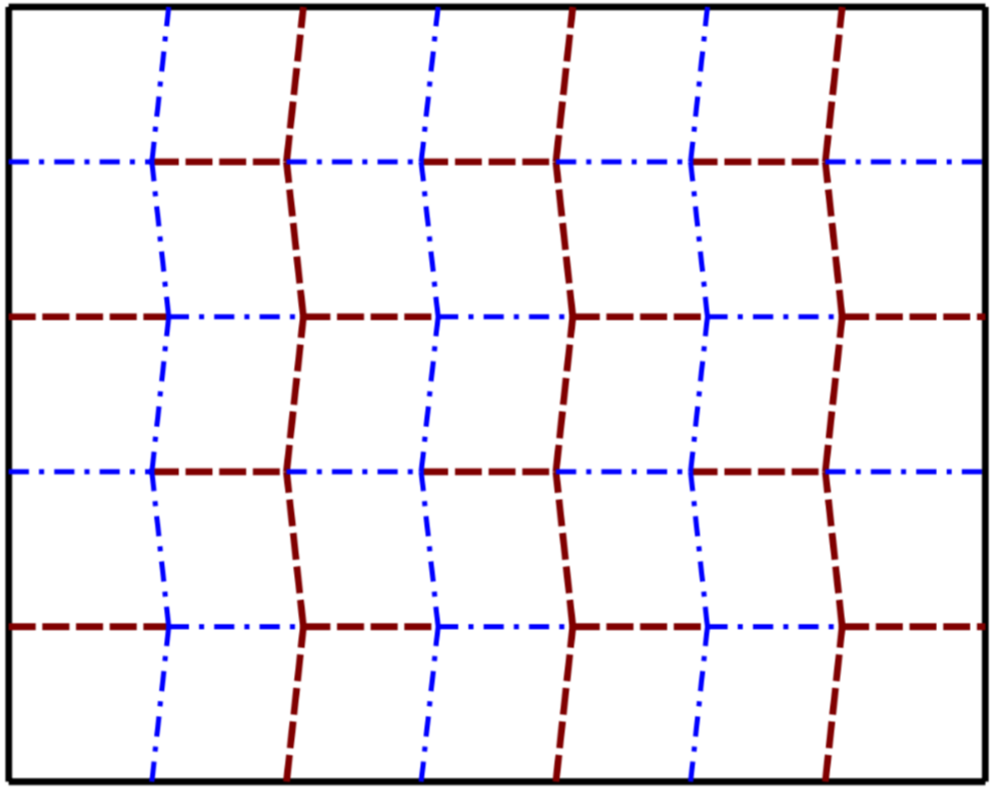

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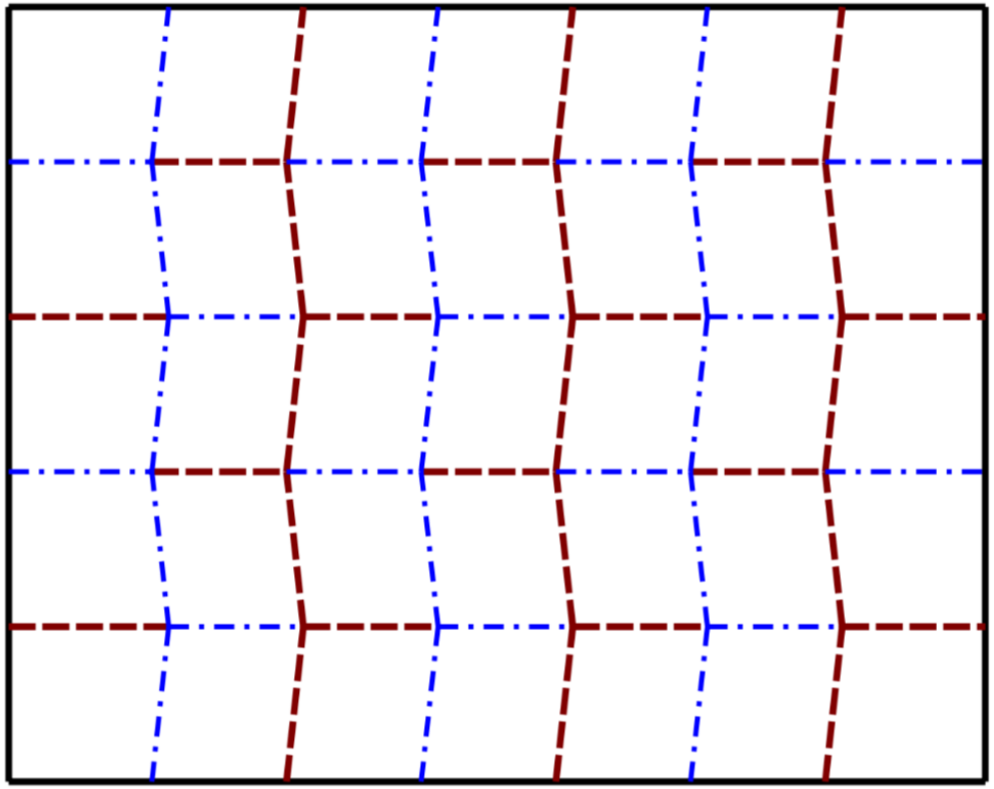
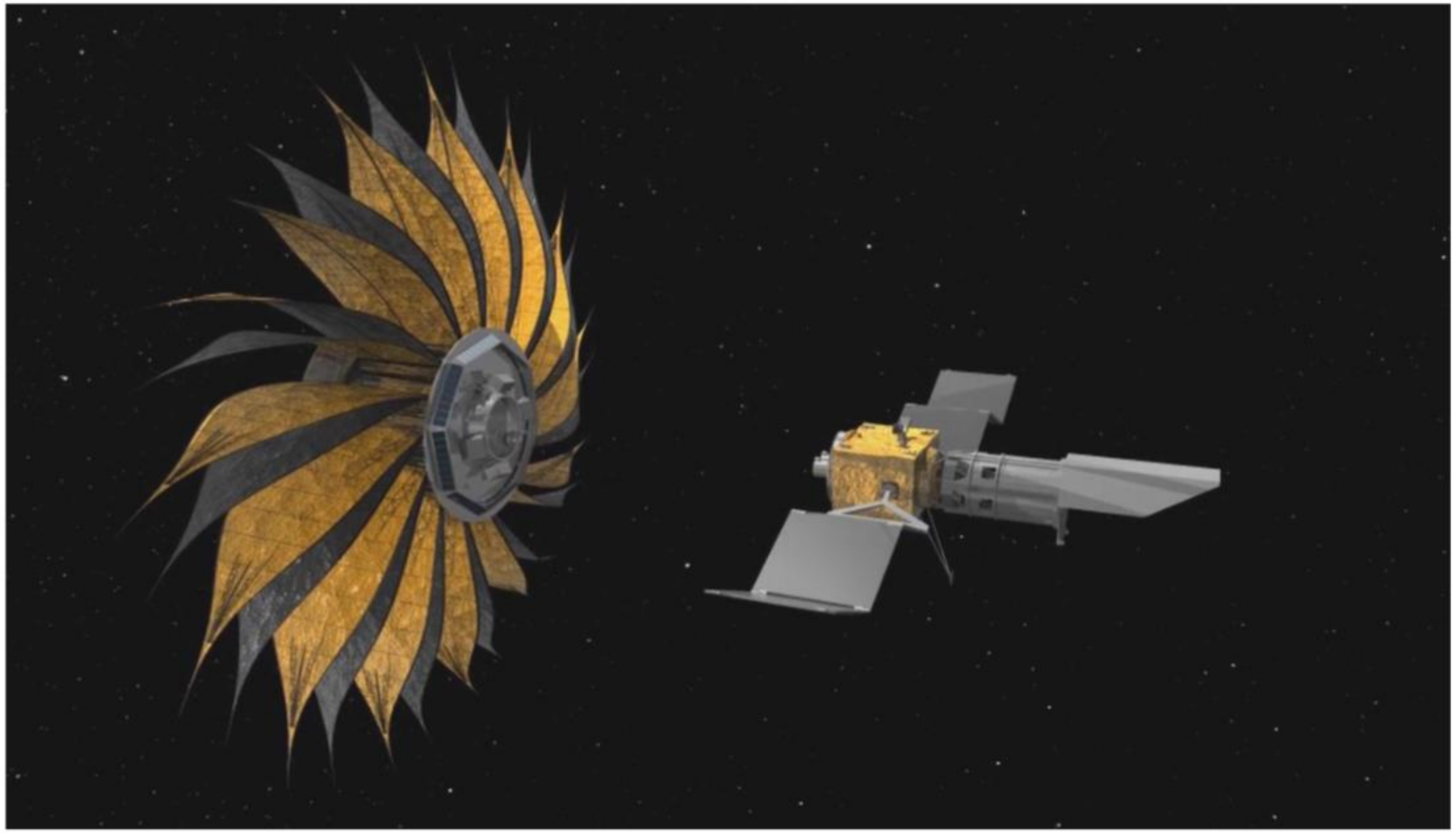
- *There is a bijection with 3-colored special graphs*



WHAT FOR?

- Starshade
- Jet Propulsion Lab



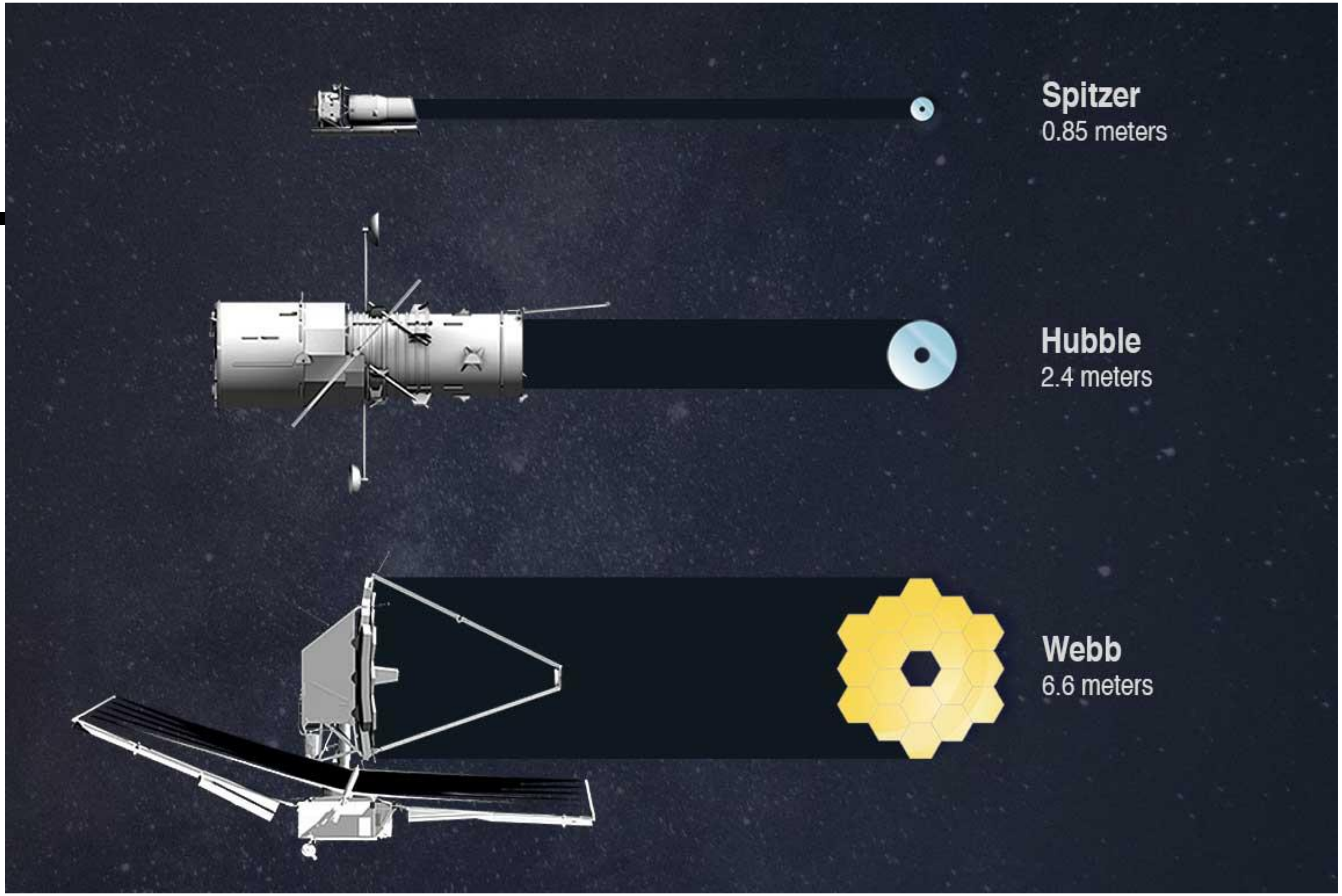
- Would block the light of the star, allowing the telescope to capture an image of the planets around the star.

WHAT FOR?

- James Webb Space Telescope

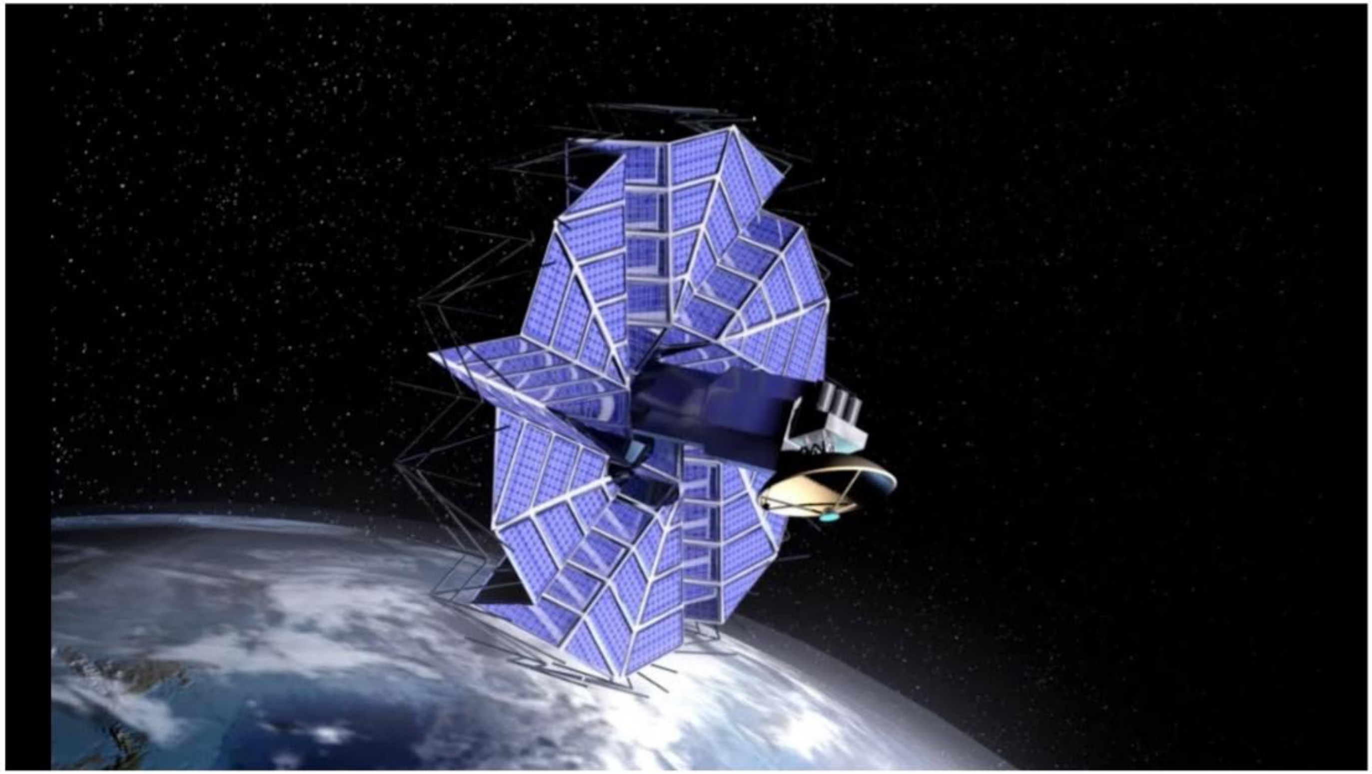
- its sunshield was folded compactly

and expanded in space



WHAT FOR?

- Eyeglass – a telescope that has 100 m in diameter origami lens



MEDICINE

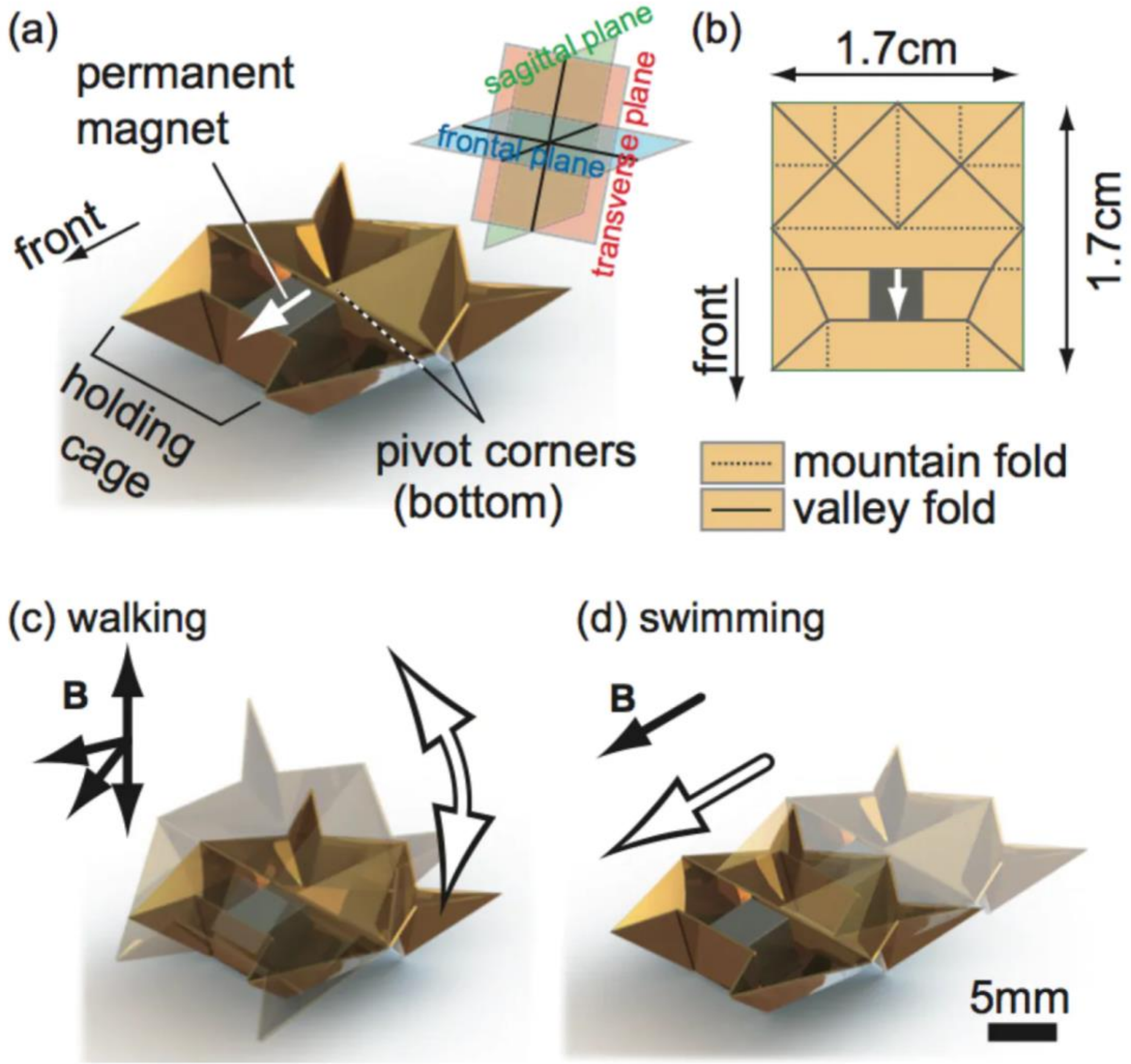
- Zhong You et al., using an origami technique water bomd, created a stent,

a tube made of mesh.

A stent is used to widen narrow or weakened blood vessels.

ORIGAMI ROBOT?

- MIT Scientists creates an origami robot which folds enough to fit into a pill. Once inside the body, it is designed to unfold on its own and move around internal organs using external magnets.



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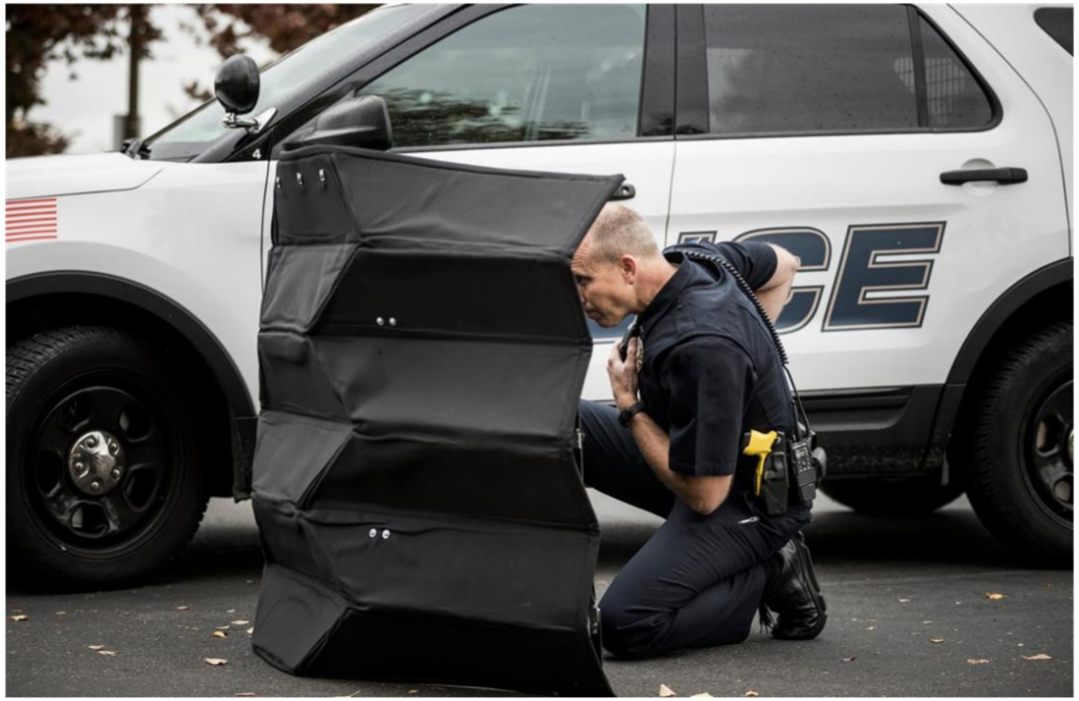
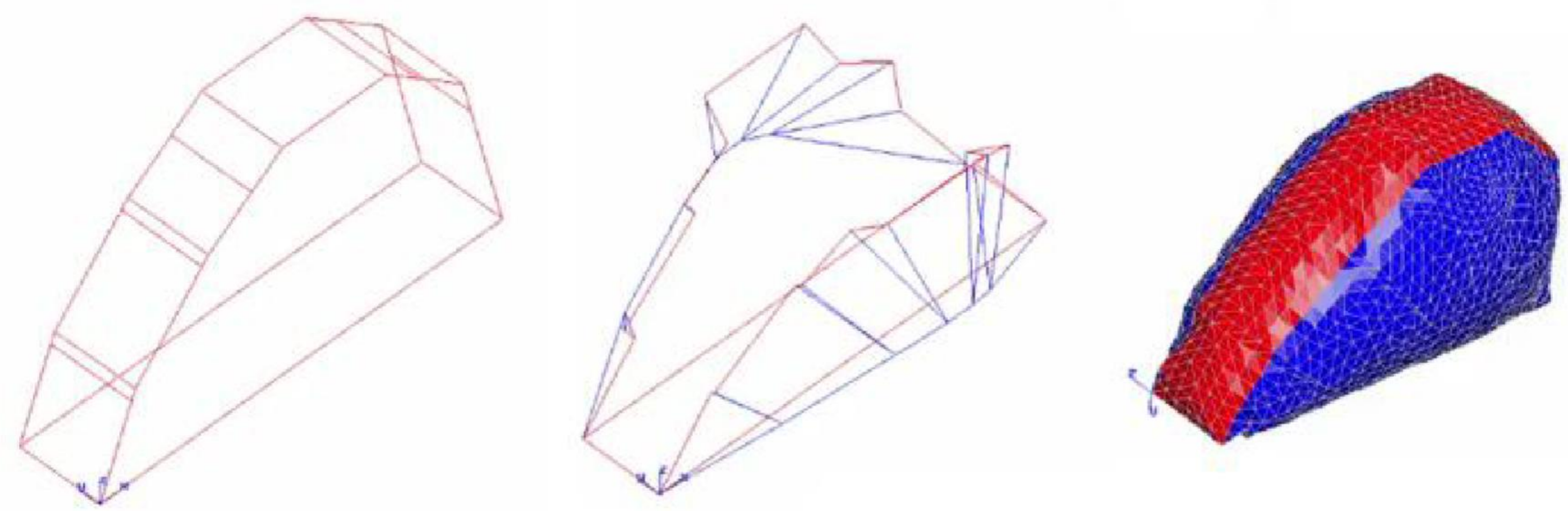


- Bulletproof screens
- Airbags
- ...

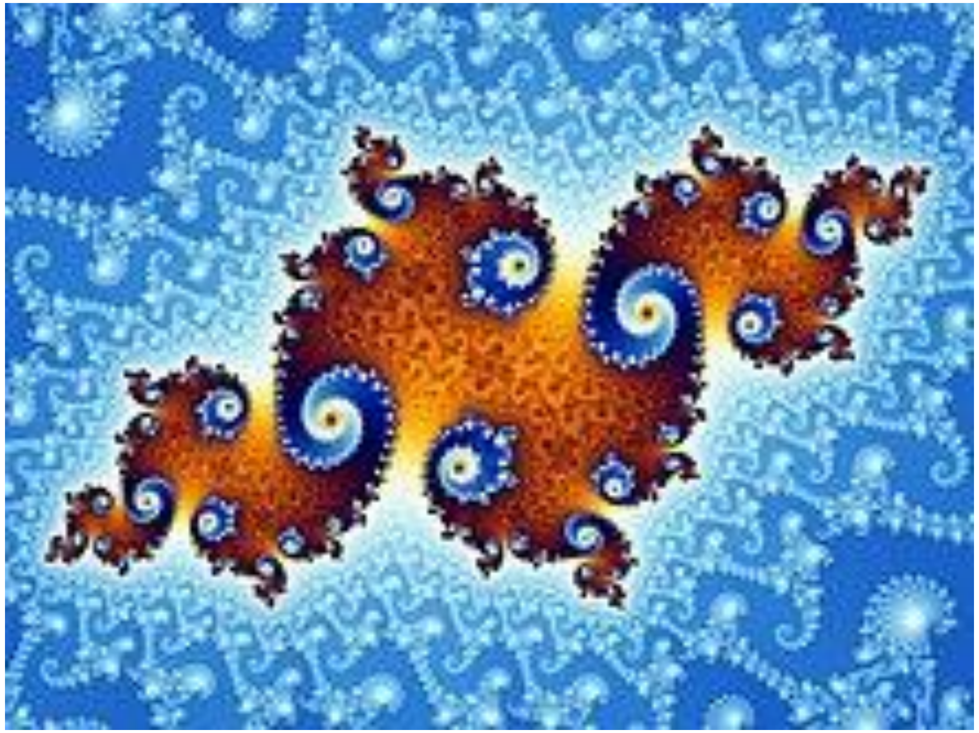


GEOMETRIES

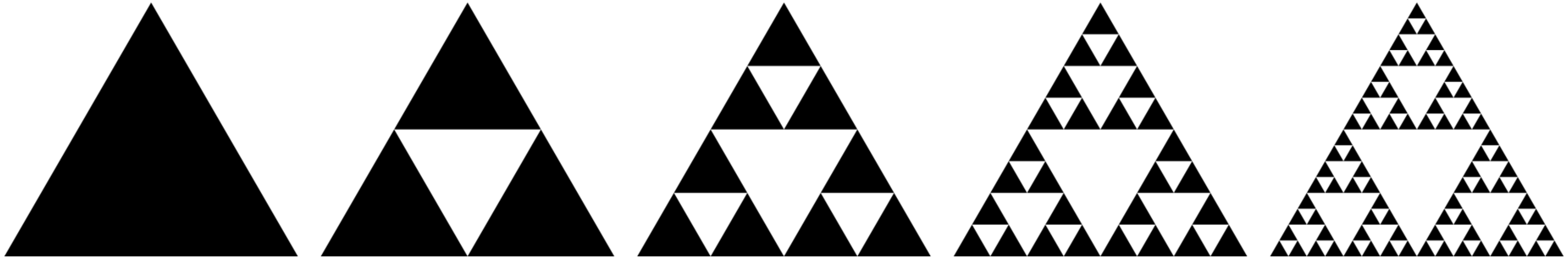
- Euclidean, Non-Euclidean, Affine, Projective, Convex, Algebraic, Discrete, Differential, Contact, Symplectic, Information, Fractal, …

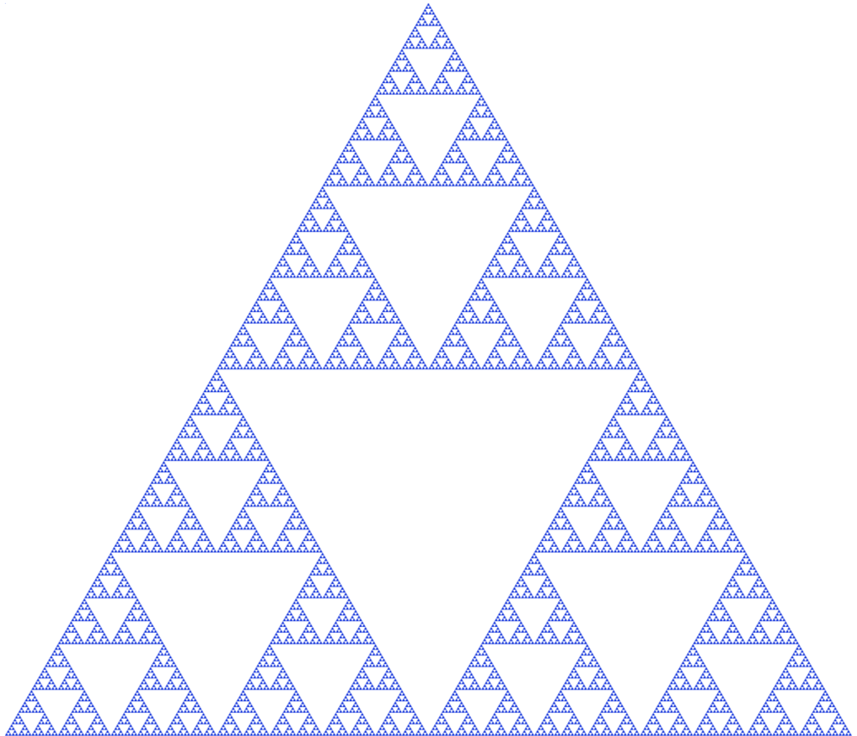
FRACTALS

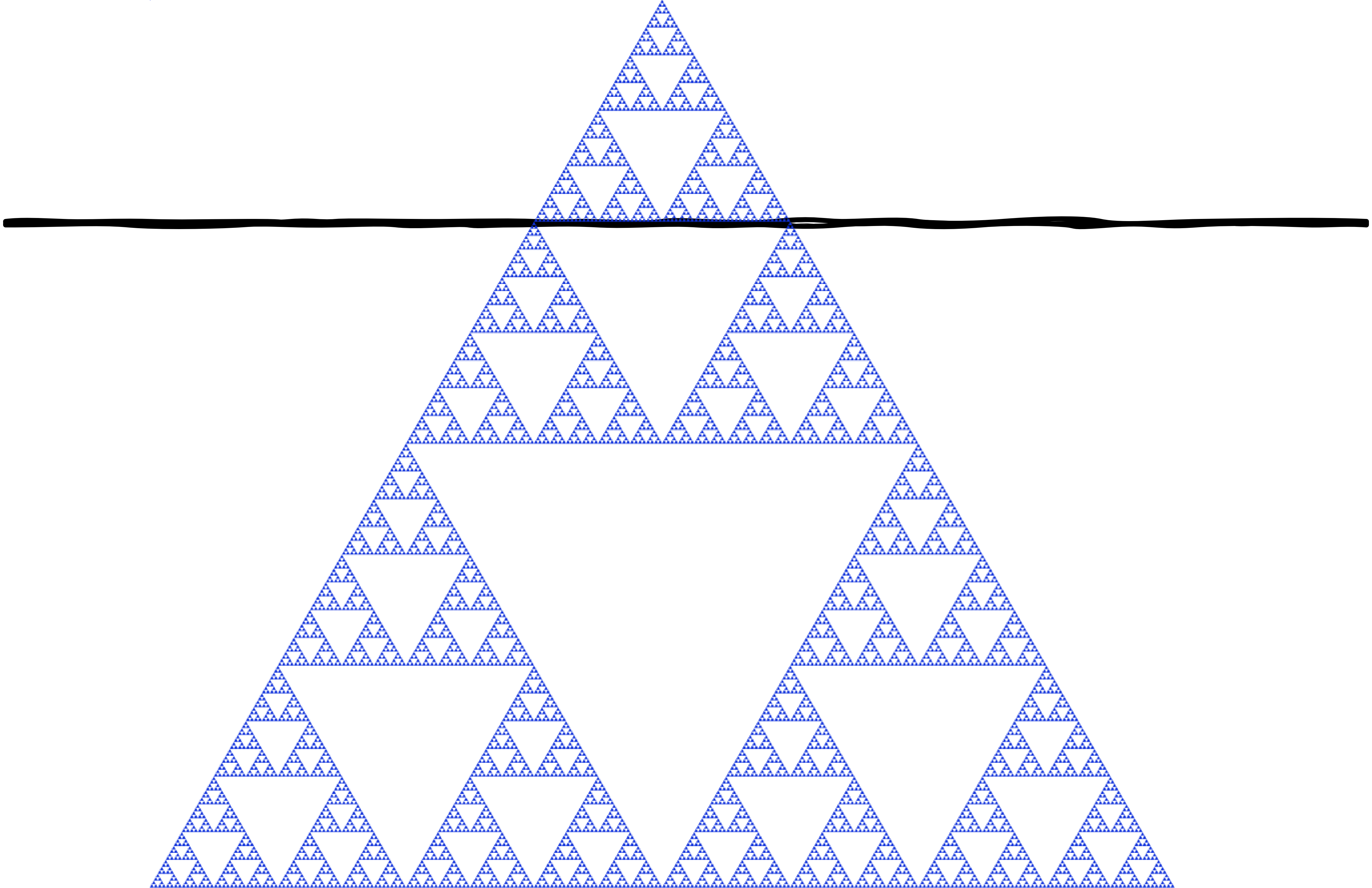
- A geometric shape that has ‚the self-similarity' property

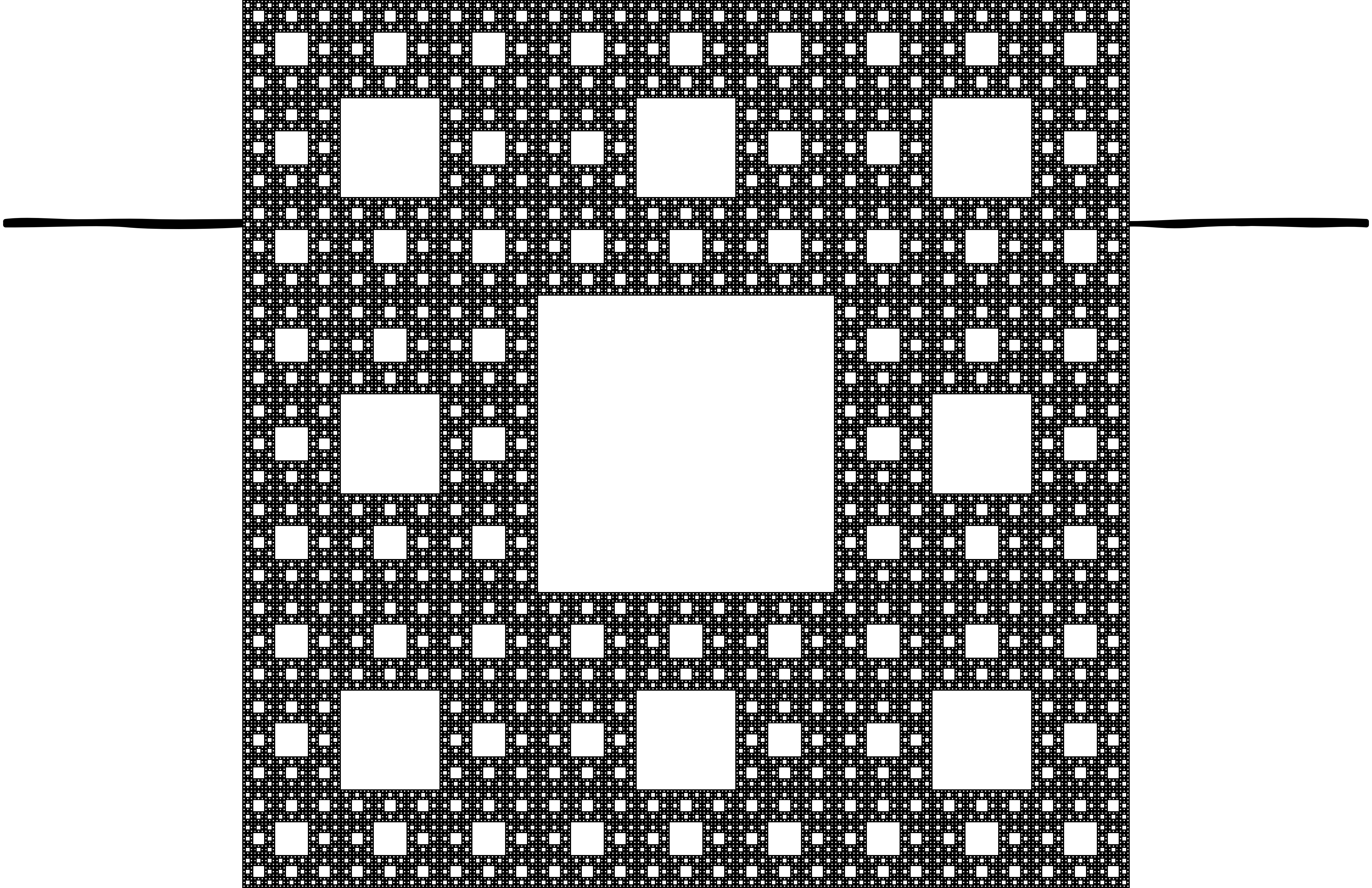


SIERPINSKI TRIANGLE

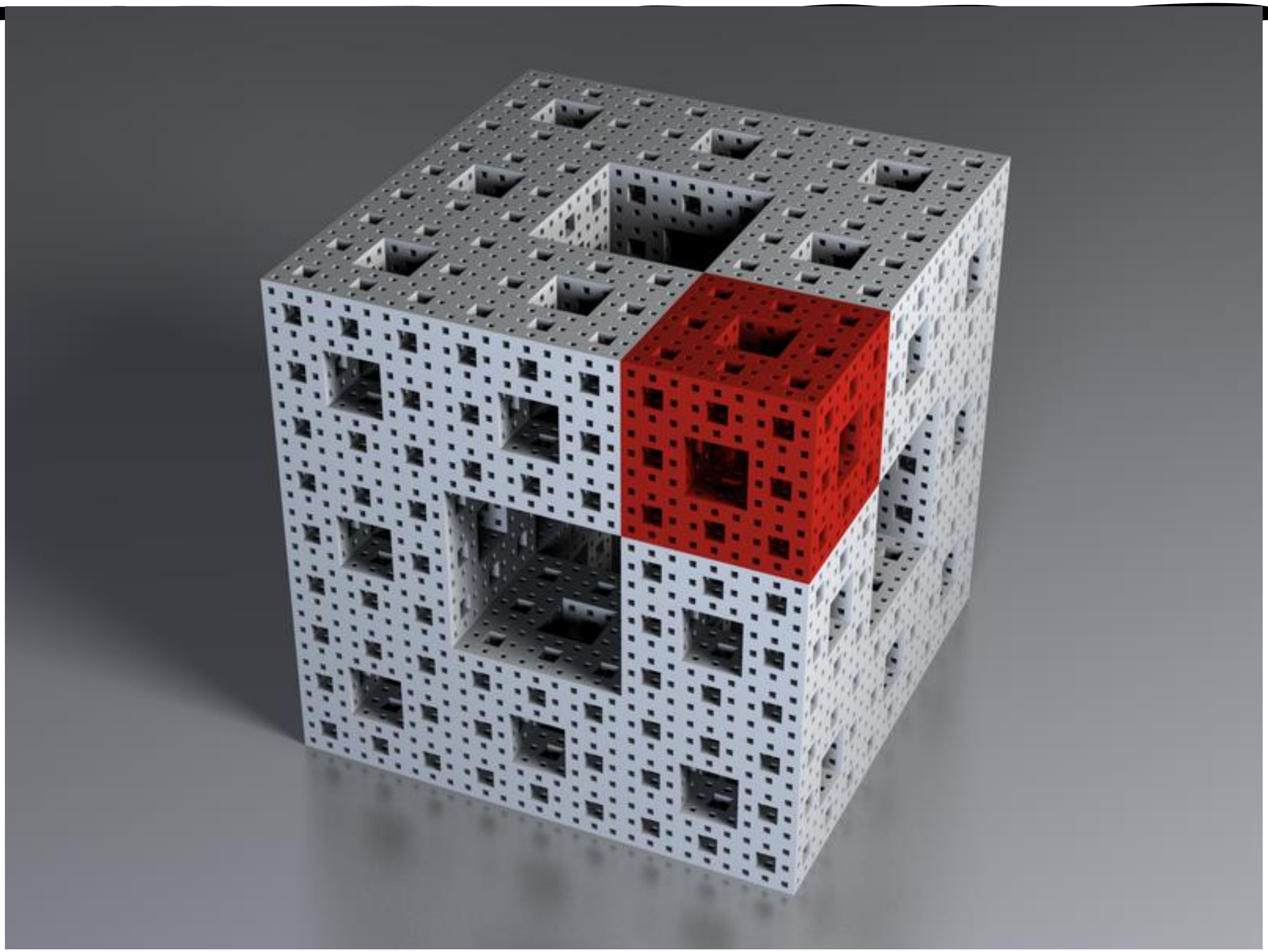




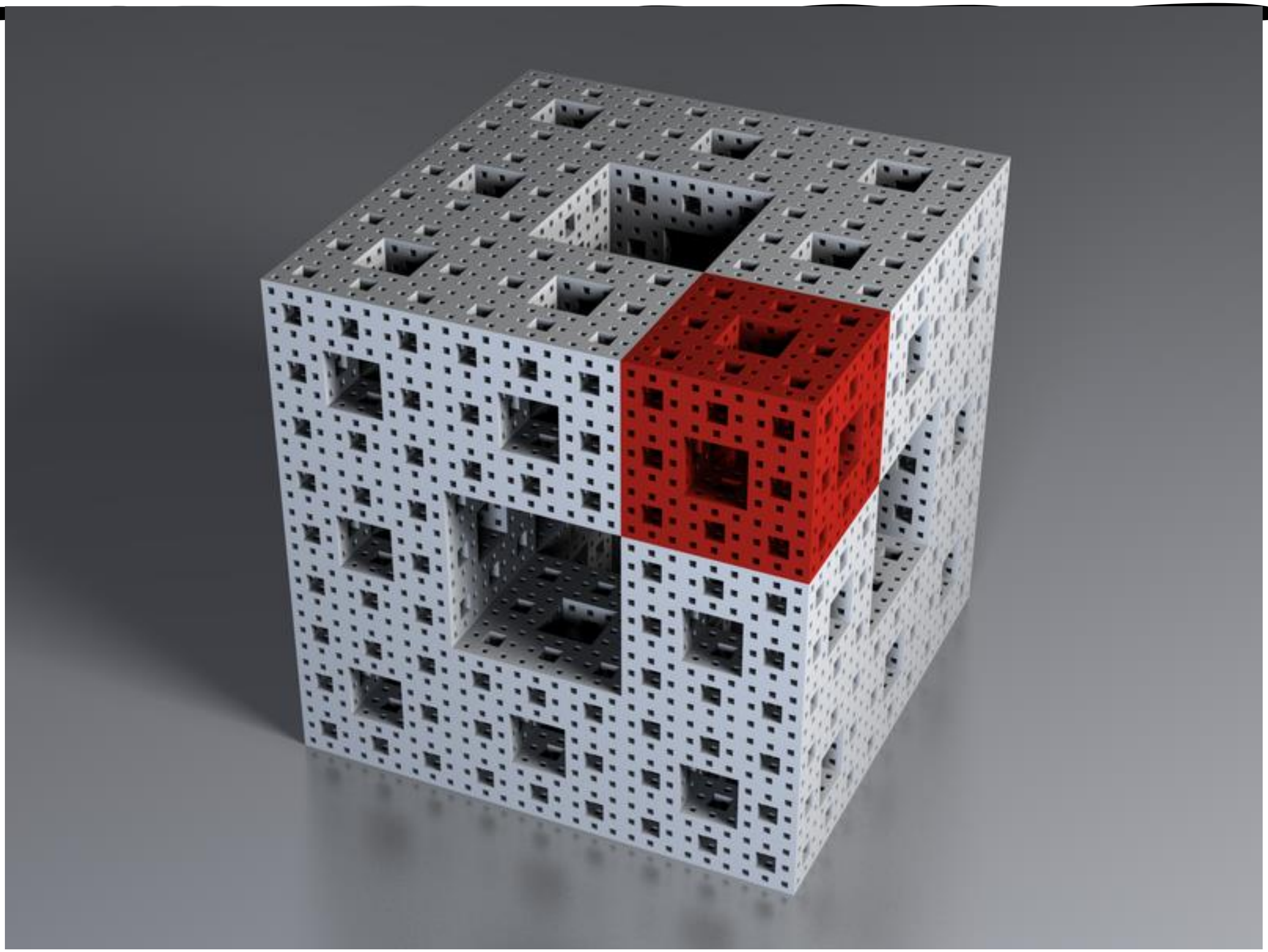




CUBE

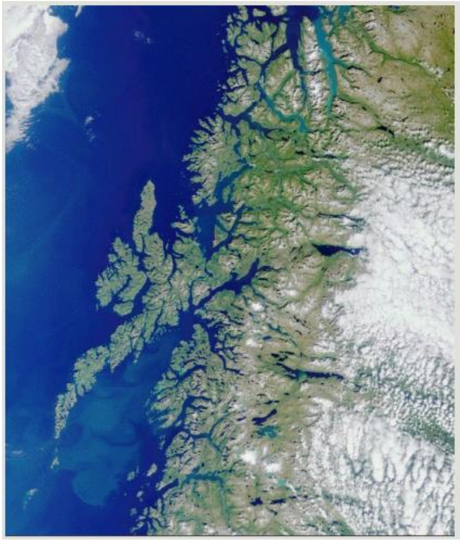


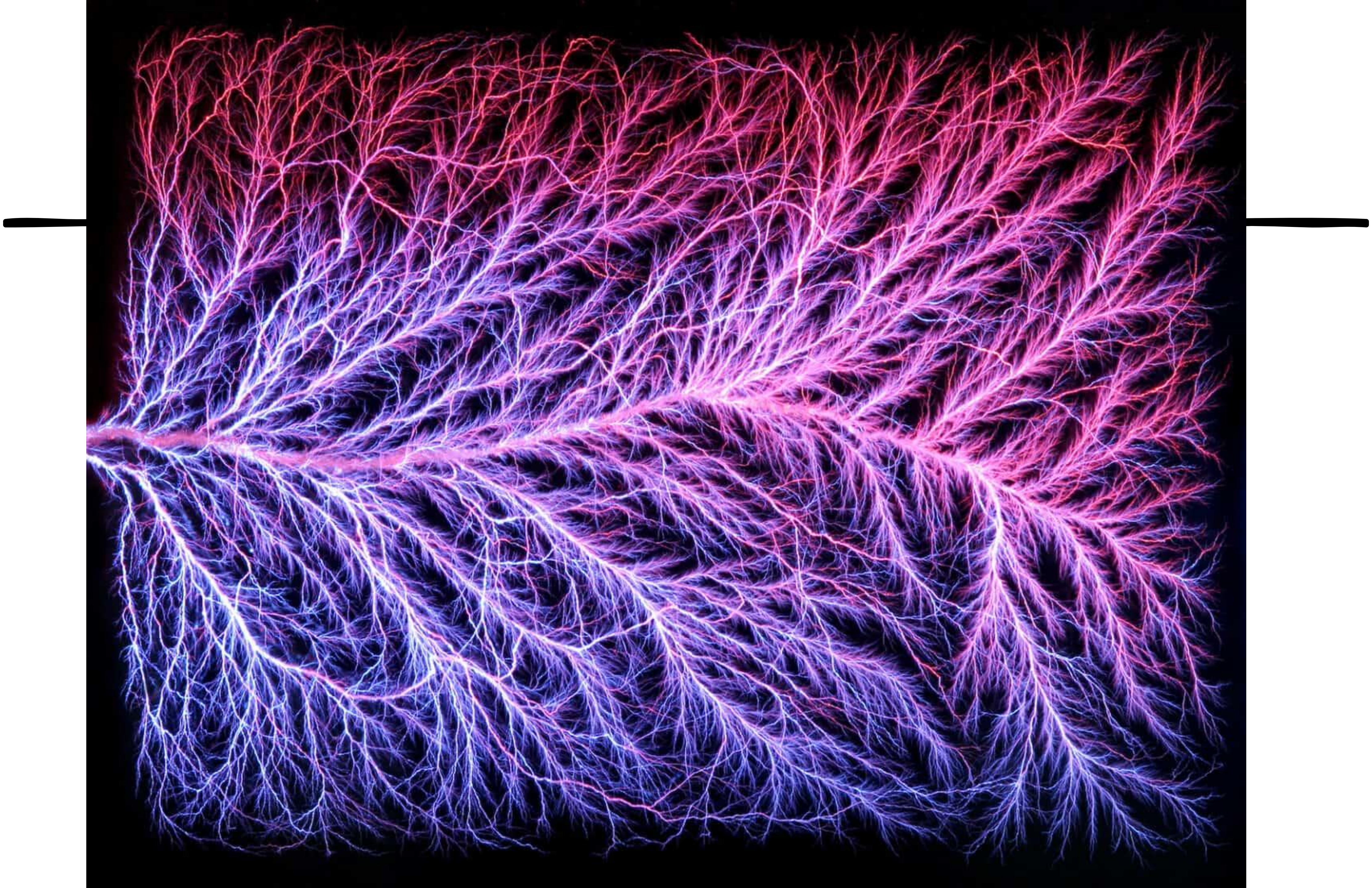
MENGER'S CUBE



FRACTALS IN NATURE



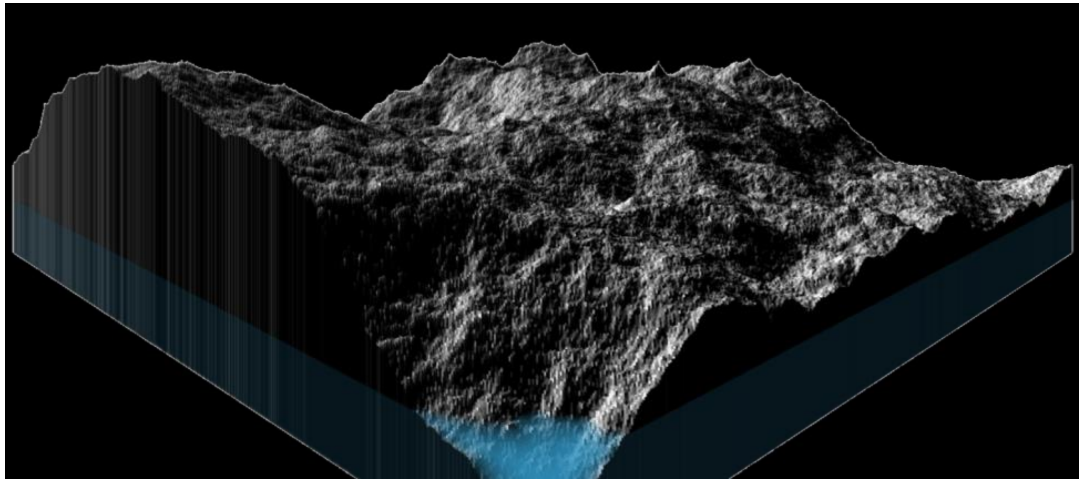


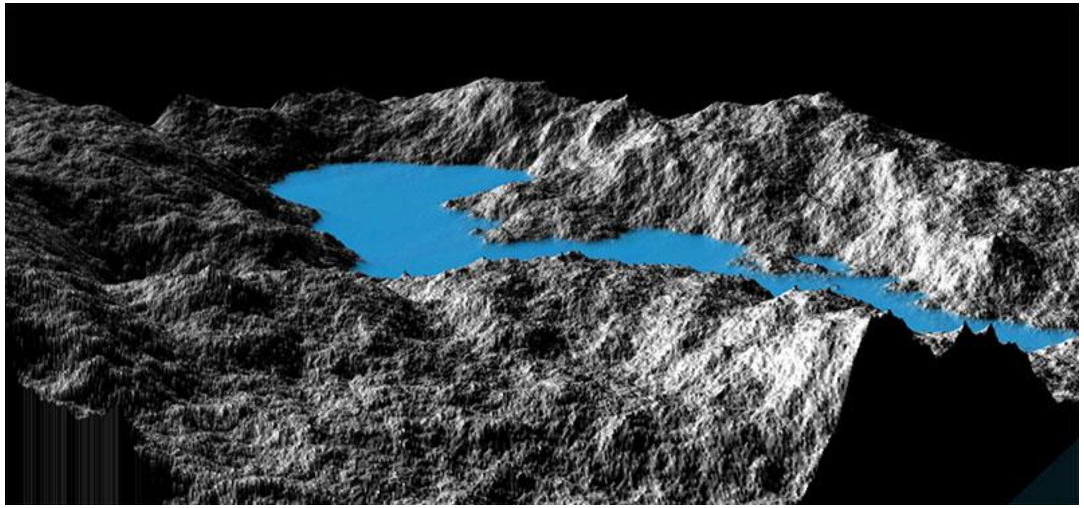



DERBY, AUSTRALIA

WHAT FOR?

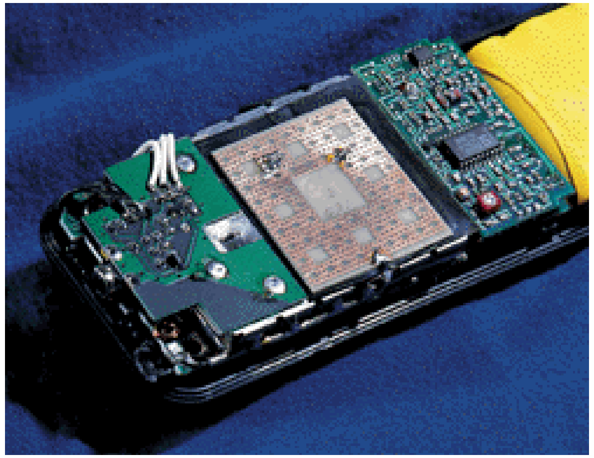
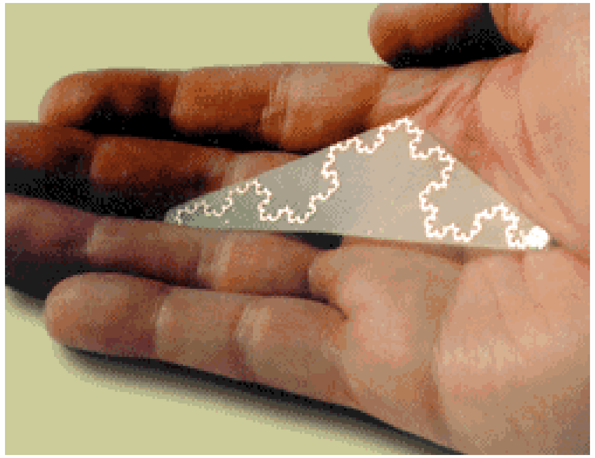
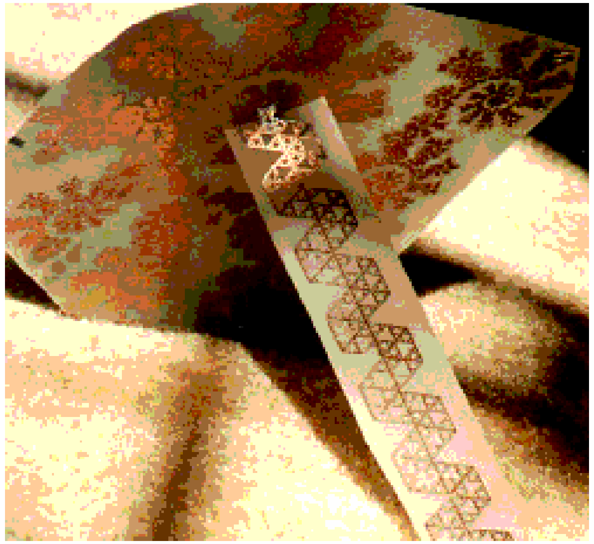
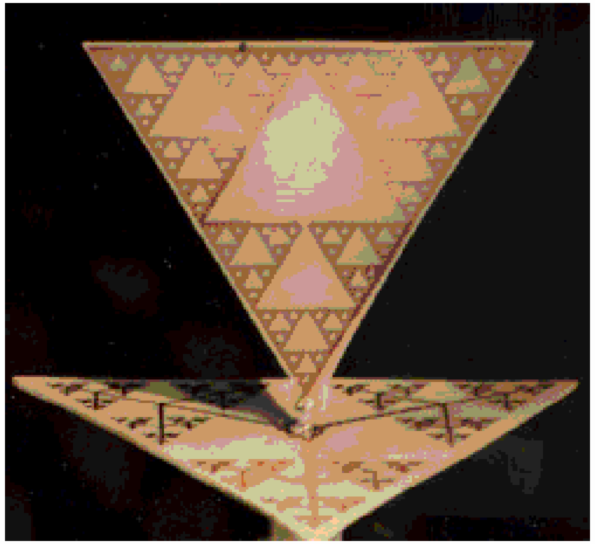
- Terrain (and other structures) generators





WHAT FOR?

- Antenas (to work properly) should have some symmetries and some selfsimilarity properties

Lindenmayer’s system (L-system)

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Then, the first iteration will give us the word „F+F",

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  - Axiom: „F”
  - Rules: „F: F+F”

Then, the first iteration will give us the word „F+F”, second: „F+F+F+F”,

Lindenmayer's system (L-system)

- It consists of three elements:
  - Alphabet
  - Starting word (axiom)
  - The rules of modifying words

- An example:
  - Alphabet: {F, +, -}
    - „F" means: „forward"
    - „+" means: „turn right α degrees"
    - „-" means: „turn left α degrees"
  - Axiom: „F"
  - Rules: „F: F+F"

Then, the first iteration will give us the word „F+F", second: „F+F+F+F", and the third: „F+F+F+F+F+F+F+F".

EXAMPLE

- Let's consider the axiom „F--F--F" and the rule: „F:F+F--F+F", and the angle of 60 degrees.

EXAMPLE

- Let's consider the axiom „F--F--F" and the rule: „F:F+F--F+F"

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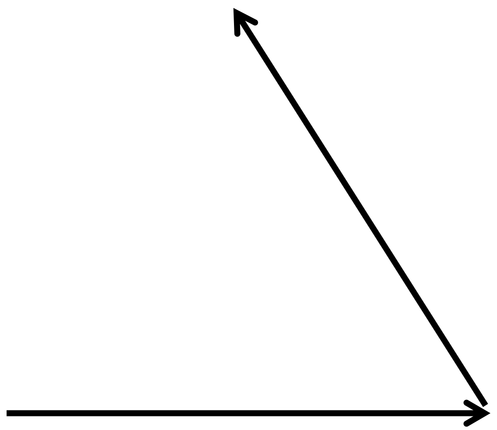

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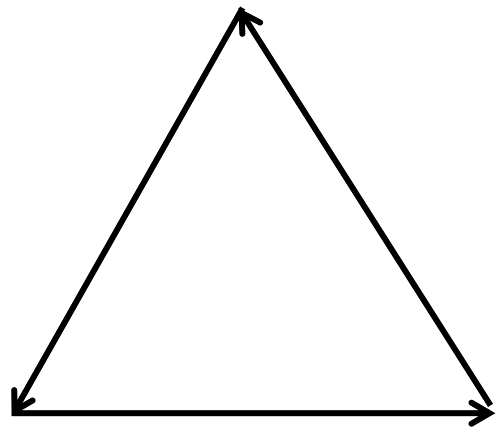

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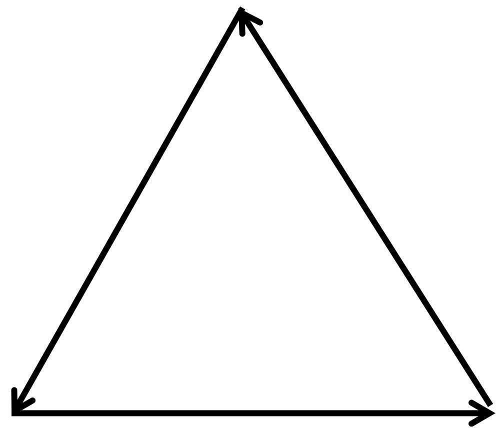

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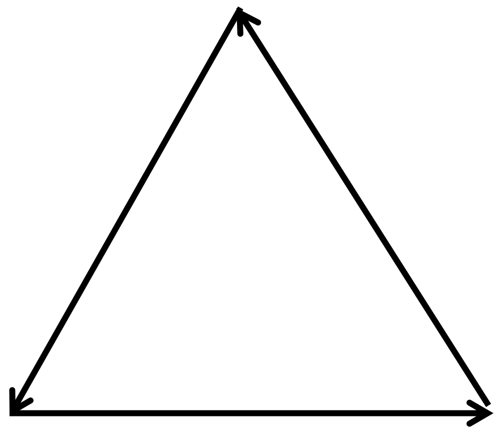

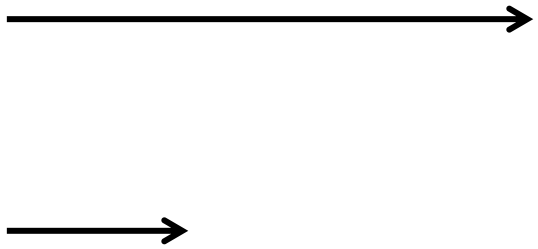

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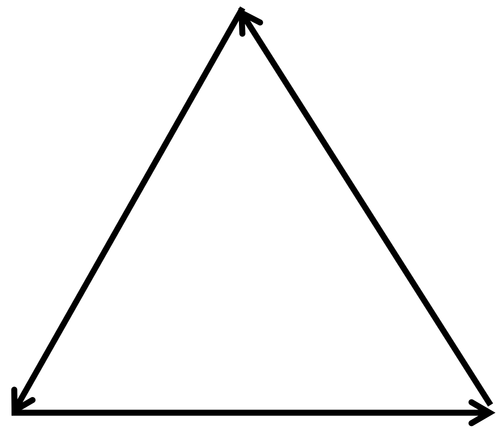

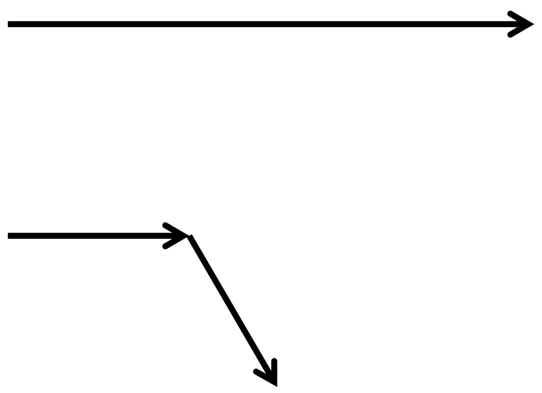

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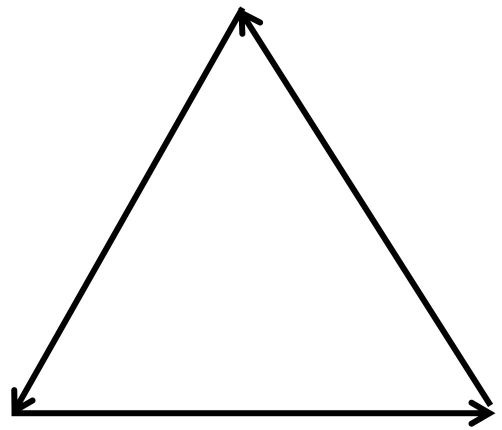

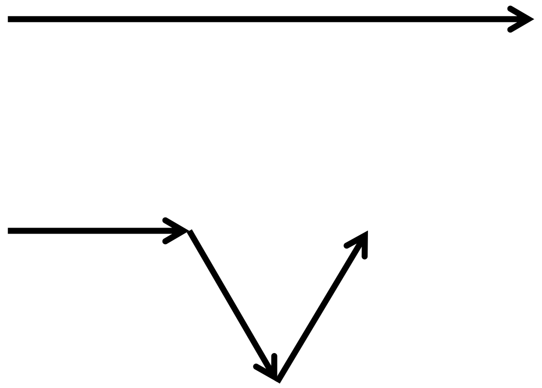

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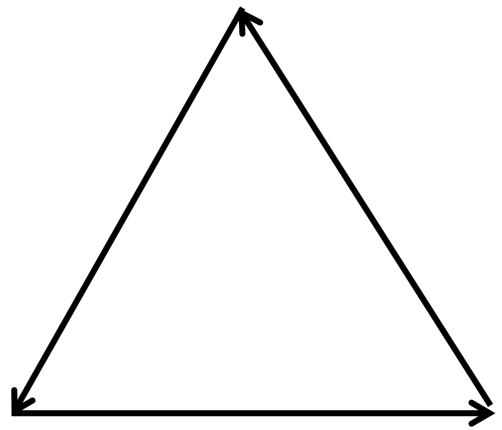

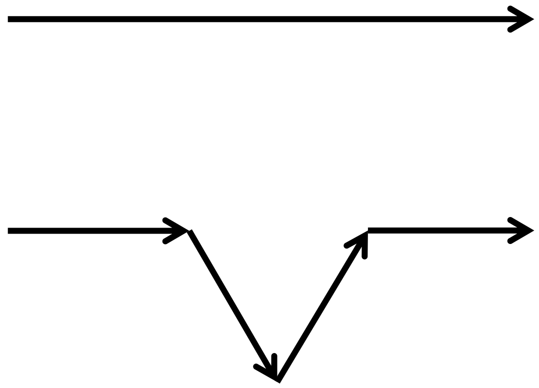

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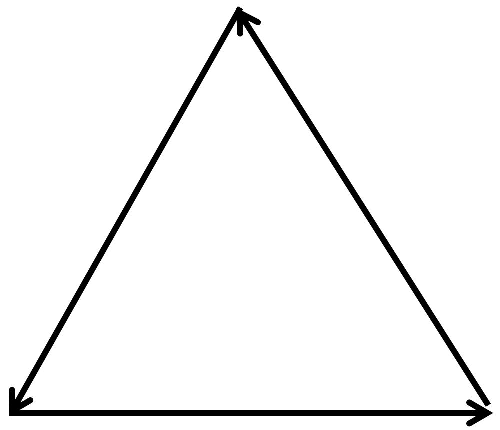
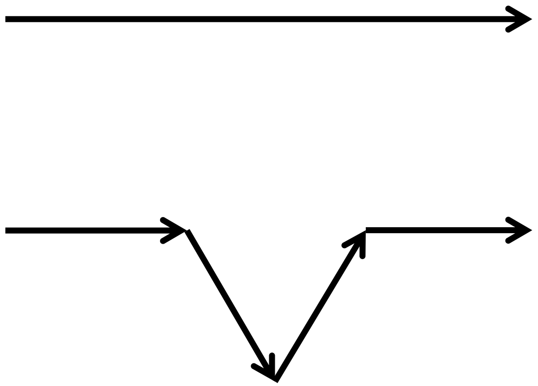
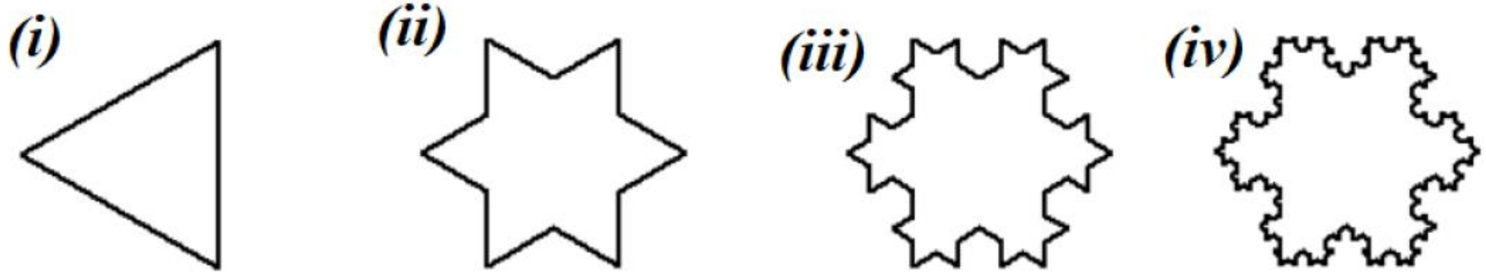
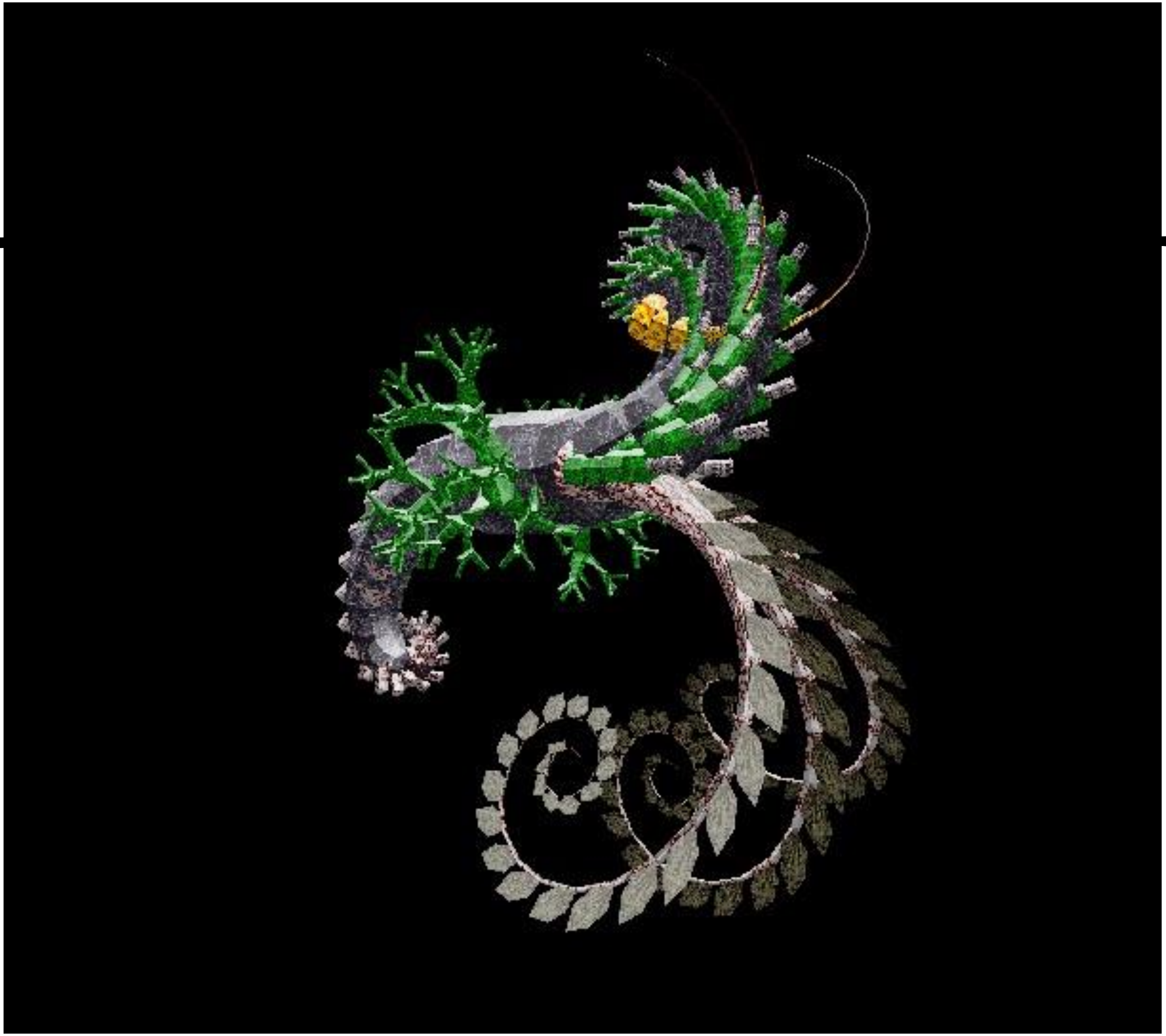
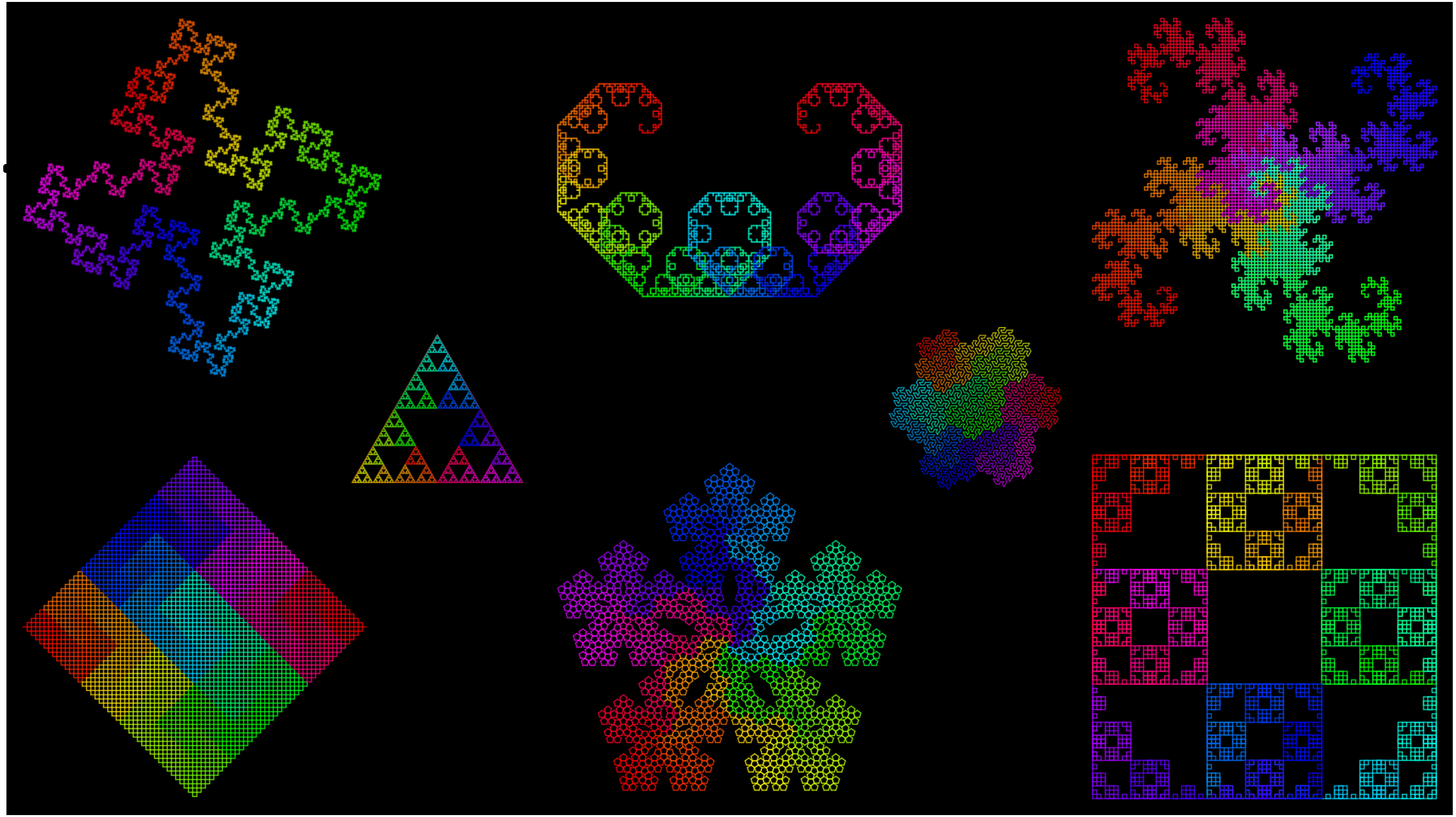


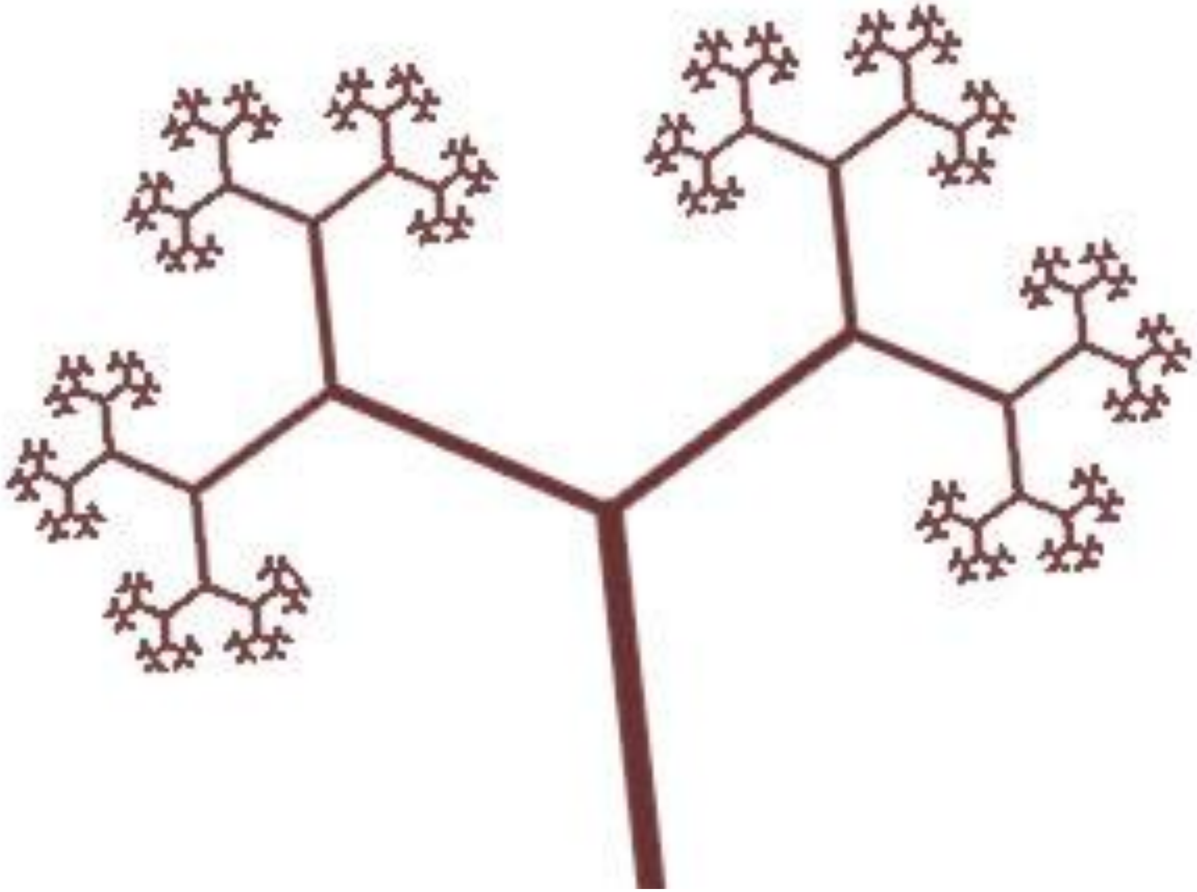




AIRHOURSE

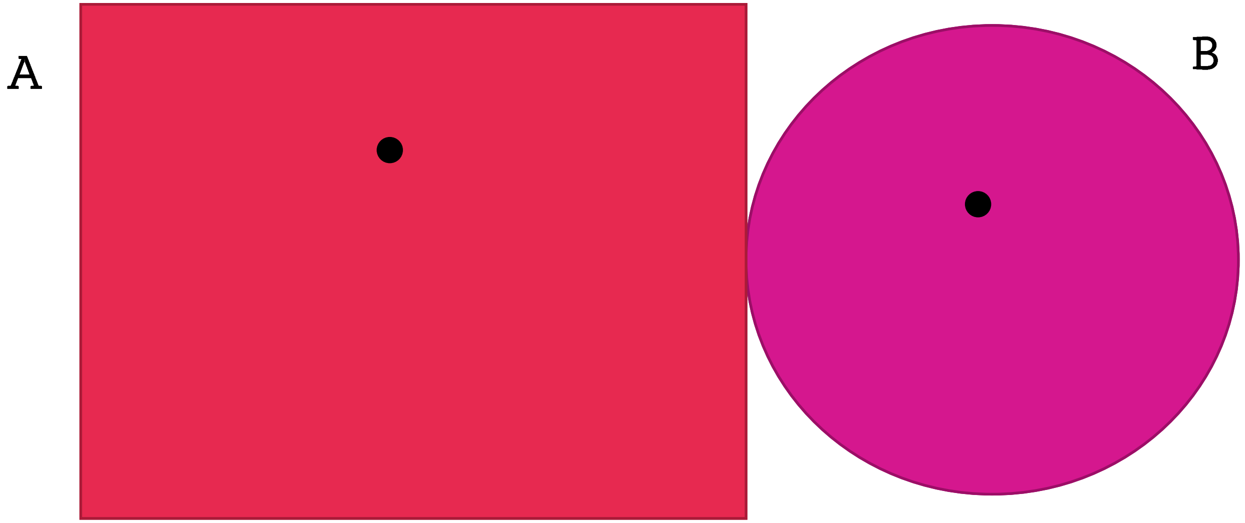






METRIC ON COMPACT SUBSPACES OF A METRIC SPACE

- Hausforff metric (cat-dog metric)
- First: we introduce a dog in A and a cat in B sets, respectively.



METRIC ON COMPACT SUBSPACES OF A METRIC SPACE

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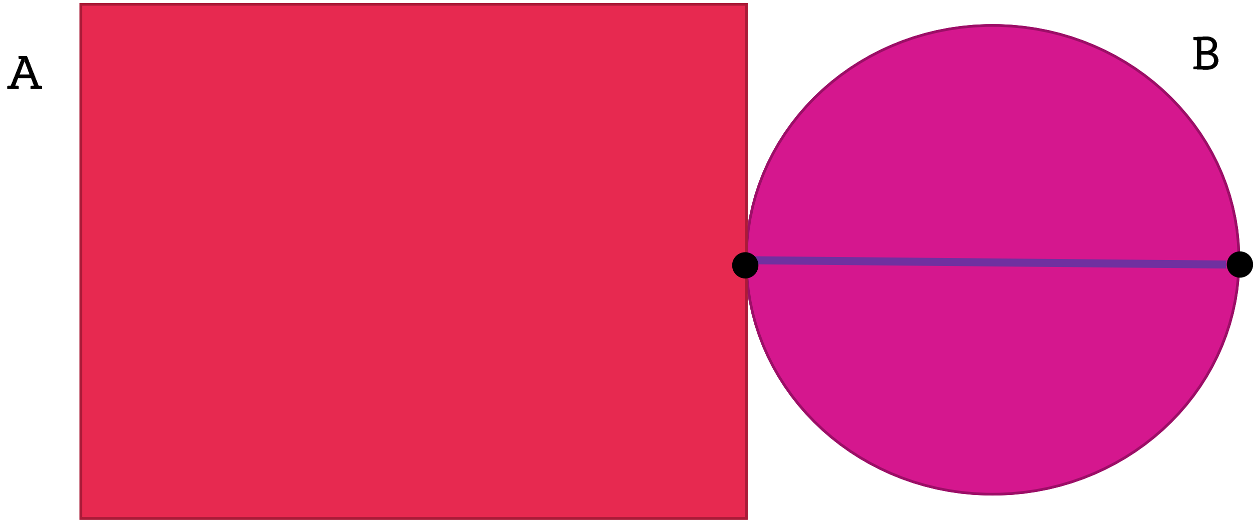

METRIC ON COMPACT SUBSPACES OF A METRIC SPACE

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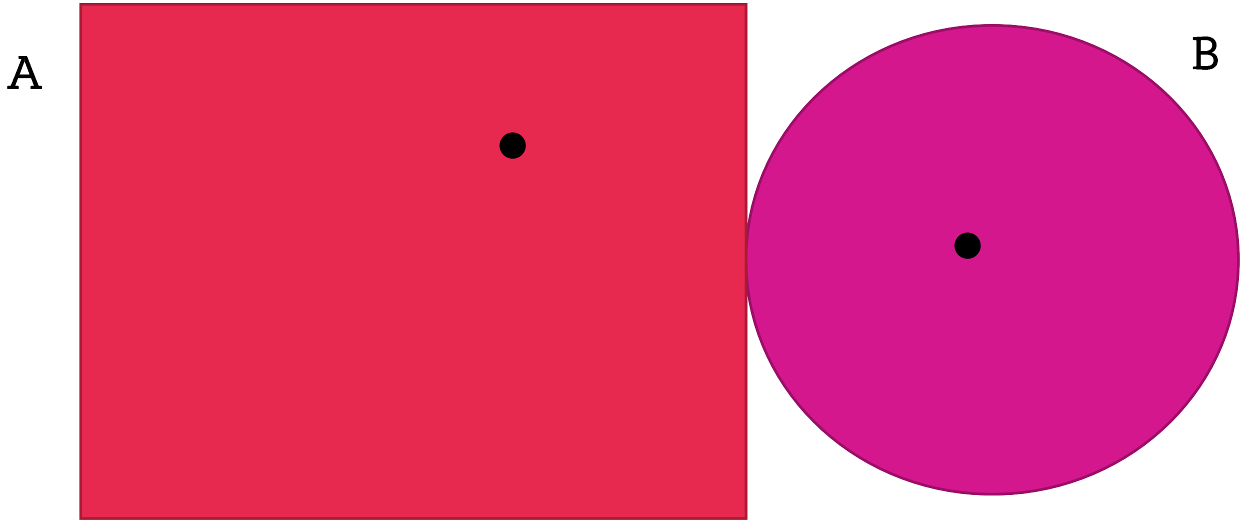
- Now, we switch them.



METRIC ON COMPACT SUBSPACES OF A METRIC SPACE

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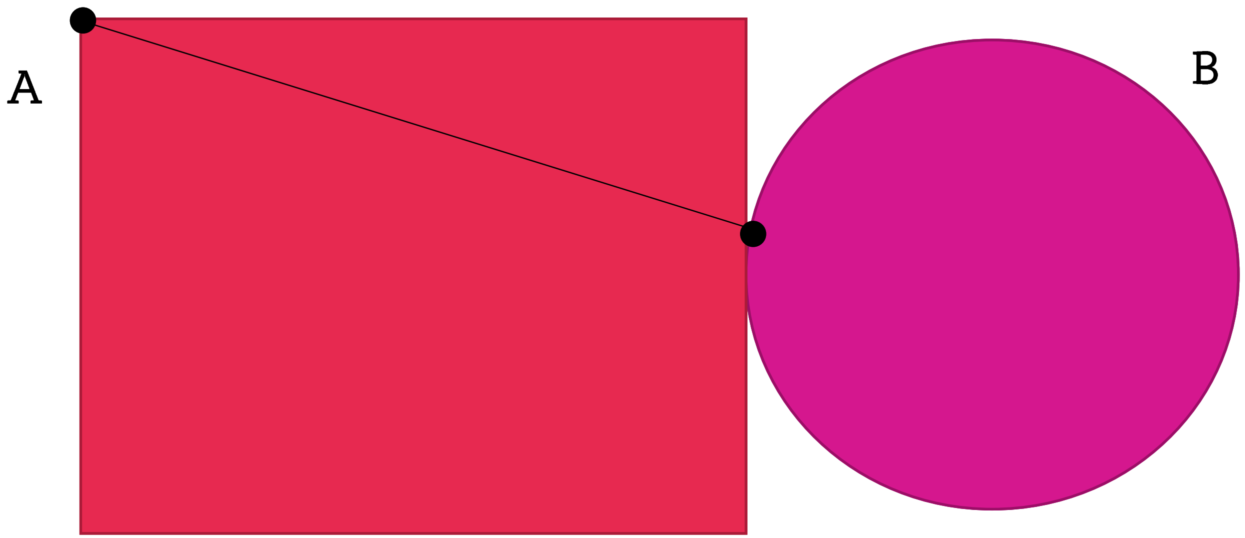
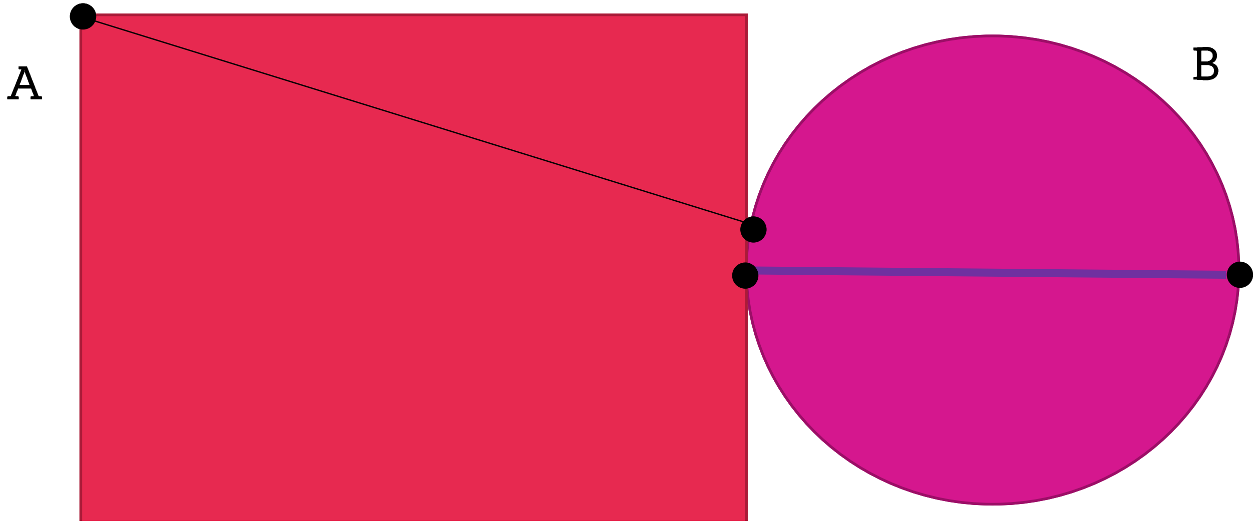


METRIC ON COMPACT SUBSPACES OF A METRIC SPACE

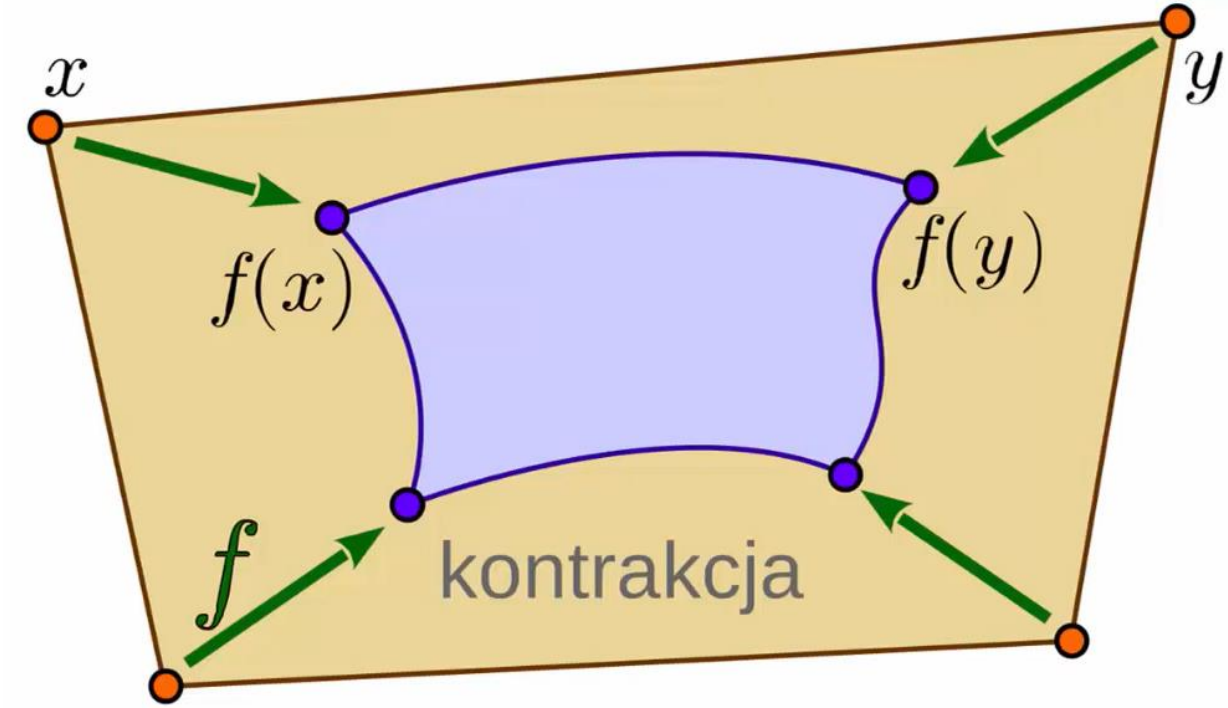
- Hausforff metric (cat-dog metric)
- Now, we switch them. Also, we write down the equilibrum distance in this case.
- Then, the Hausdorff distance is the smallest of these two numbers.



$$d_H(X,Y)=max\{d_{XY},d_{YX}\}=max\left\{\max_{x\in X}\min_{y\in Y} d(x,y),\max_{y\in Y}\min_{x\in X} d(x,y)\right\}$$

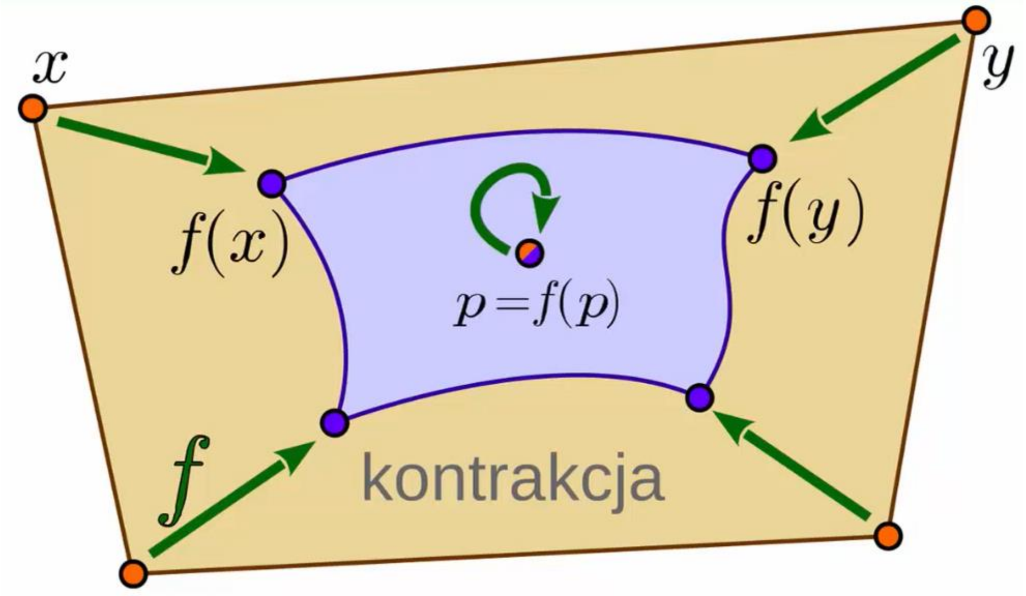
CONTRACTION

- Let (X, d) be a metric space. Then, a function $f: X \to X$ is a contraction when there exist a constant $C < 1$ for which $d\big(f(x), f(y)\big) < Cd(x, y)$.



BANACH FIXED-POINT THEOREM

- If (X, d) is a complete metric space, and $f: X \to X$ is a contraction, then there exists the unique point p such that $f(p) = p$.



Furthermore, $p = \lim_{n} f^{n}(x)$.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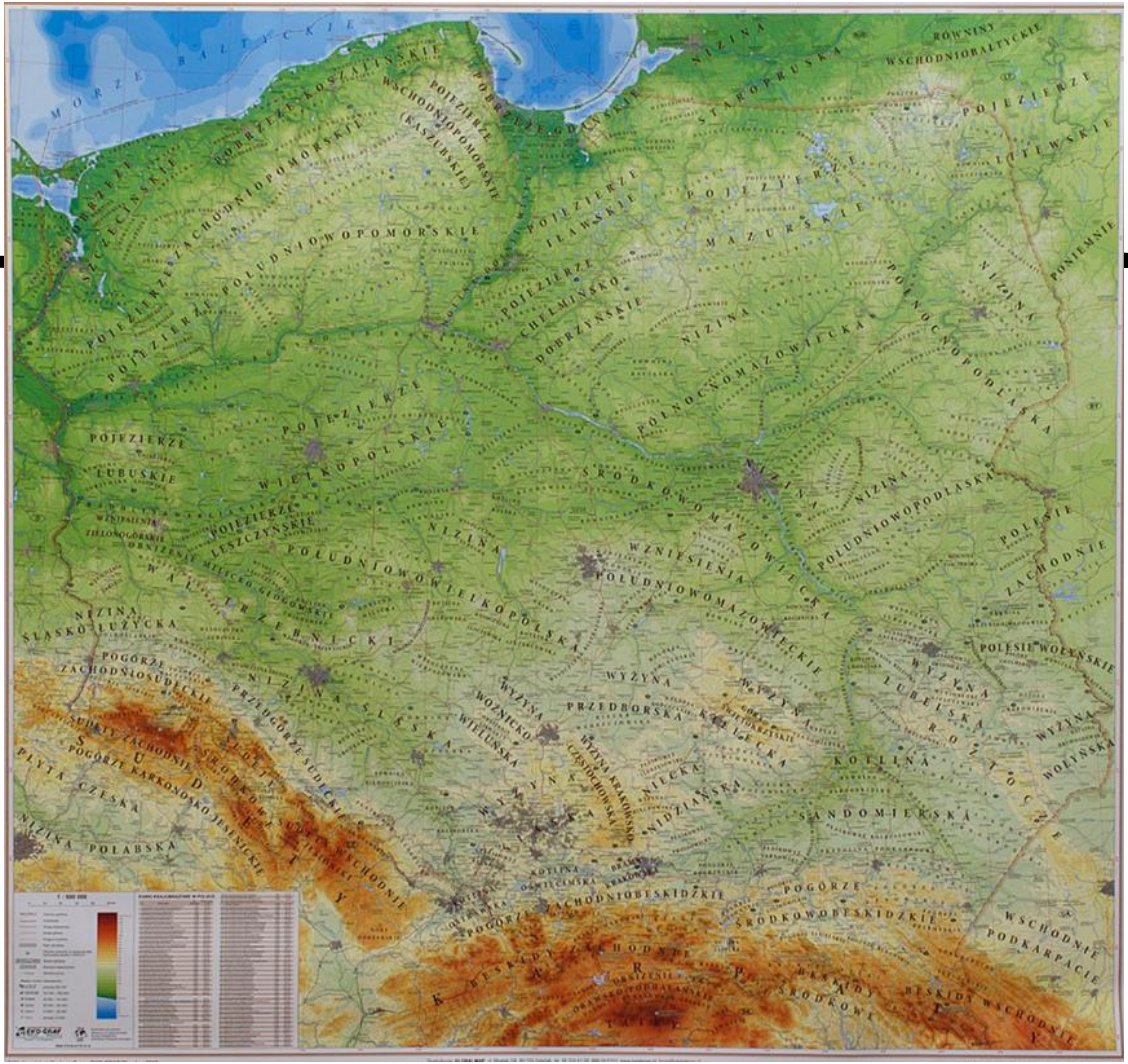

MORZE BAŁTYCKIE
NIZINA STAROPRUSKA
RÓWNINY WSCHODNIOBAŁTYCKIE
POJEZIERZE LITEWSKIE
POBRZEŻE KOSZALIŃSKIE
POBRZEŻE GDAŃSKIE
POJEZIERZE WSCHODNIOPOMORSKIE (KASZUBSKIE)
POBRZEŻE SZCZECIŃSKIE
POJEZIERZE ZACHODNIOPOMORSKIE
POJEZIERZE POŁUDNIOWOPOMORSKIE
POJEZIERZE IŁAWSKIE
POJEZIERZE MAZURSKIE
POJEZIERZE CHEŁMIŃSKO-DOBRZYŃSKIE
NIZINA PÓŁNOCNOMAZOWIECKA
NIZINA PÓŁNOCNOPODLASKA
POJEZIERZE WIELKOPOLSKIE
POJEZIERZE LUBUSKIE
NIZINA ŚRODKOWOMAZOWIECKA
NIZINA POŁUDNIOWOPODLASKA
POLESIE ZACHODNIE
POJEZIERZE LESZCZYŃSKIE
NIZINA POŁUDNIOWOWIELKOPOLSKA
WZNIESIENIA POŁUDNIOWOMAZOWIECKIE
WAŁ TRZEBNICKI
NIZINA ŚLĄSKO-ŁUŻYCKA
POLESIE WOŁYŃSKIE
POGÓRZE ZACHODNIOSUDECKIE
NIZINA ŚLĄSKA
PRZEDGÓRZE SUDECKIE
WYŻYNA WOŹNICKO-WIELUŃSKA
WYŻYNA PRZEDBORSKA
WYŻYNA KIELECKA
WYŻYNA LUBELSKA
ROZTOCZE
WYŻYNA WOŁYŃSKA
WYŻYNA KRAKOWSKO-CZĘSTOCHOWSKA
NIECKA NIDZIAŃSKA
KOTLINA SANDOMIERSKA
WYŻYNA ŚLĄSKA
SUDETY
SUDETY ZACHODNIE
SUDETY ŚRODKOWE
SUDETY WSCHODNIE
PŁYTA CZESKA
POGÓRZE KARKONOSKO-JESENICKIE
NIZINA POLABSKA
KOTLINA OŚWIĘCIMSKA
POGÓRZE ZACHODNIOBESKIDZKIE
POGÓRZE ŚRODKOWOBESKIDZKIE
WSCHODNIE PODKARPACIE
KARPATY
BESKIDY ZACHODNIE
BESKIDY ŚRODKOWE
BESKIDY WSCHODNIE
TATRY

THEOREM

- If (X, d) is complete metric space, then the space of all compact subsets, with Hausdorff metric, is also complete.

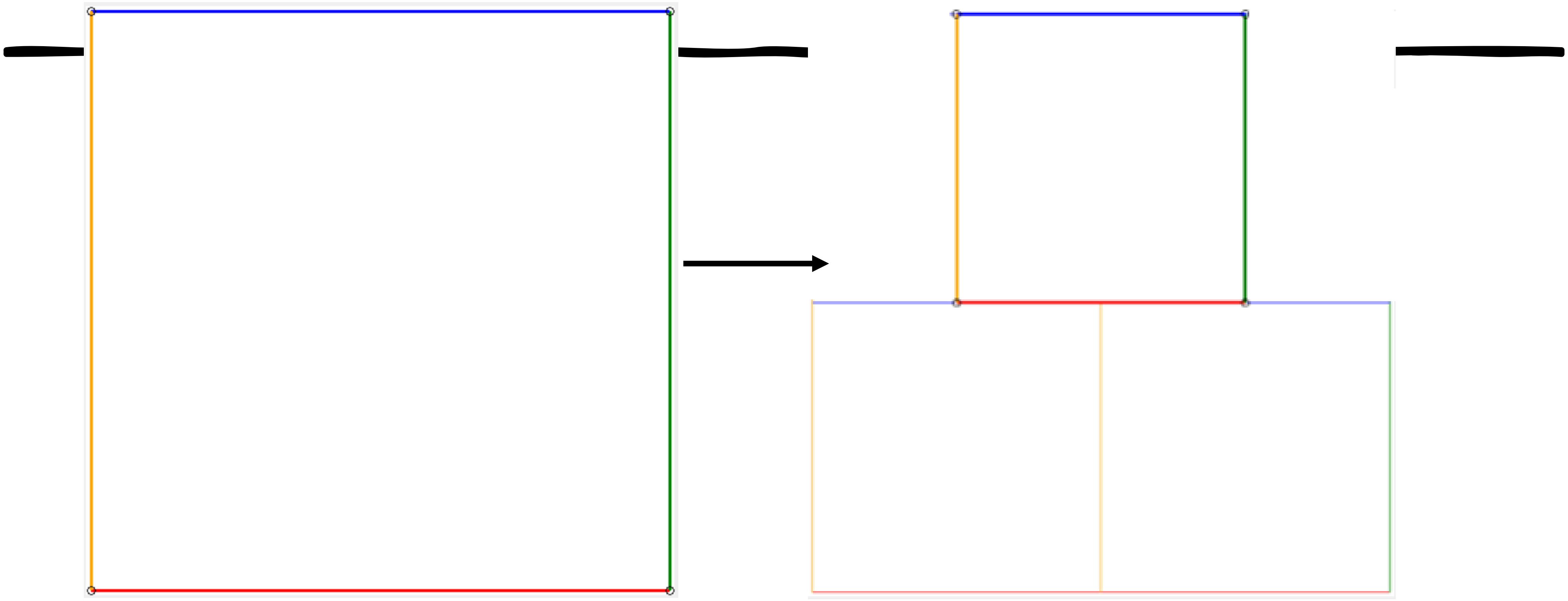
- So we can apply Banach fixed-point theorem, if we can find some contractions.

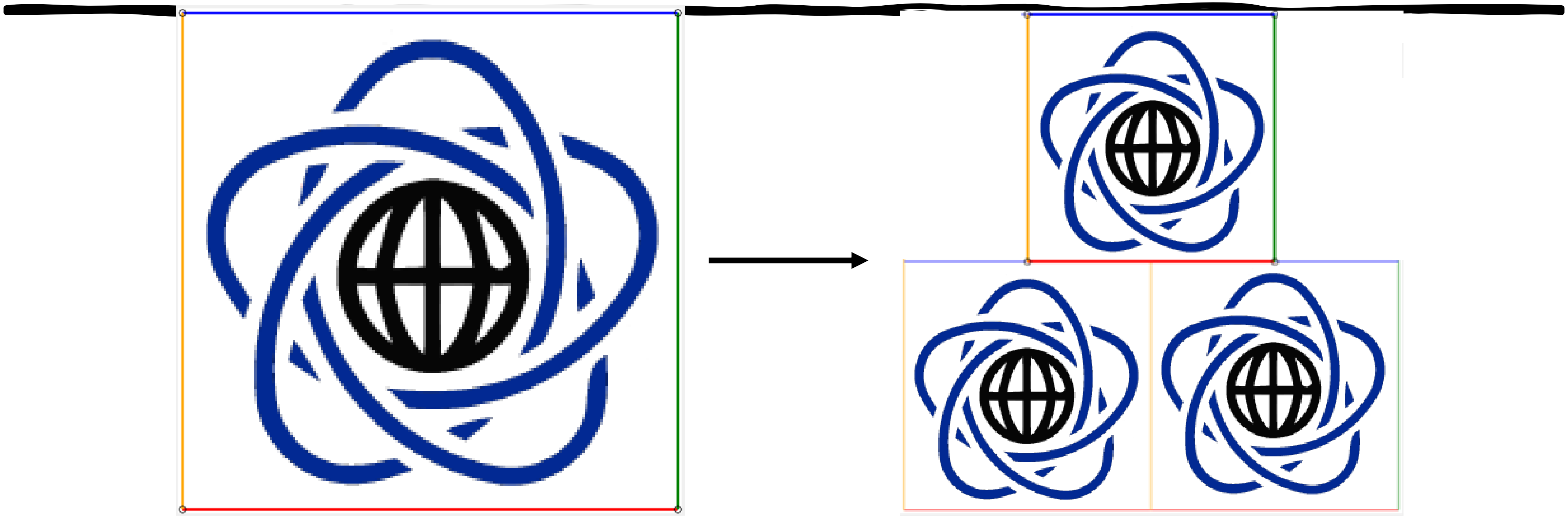
MANY CONTRACTIONS (HUTCHINSON OPERATOR)

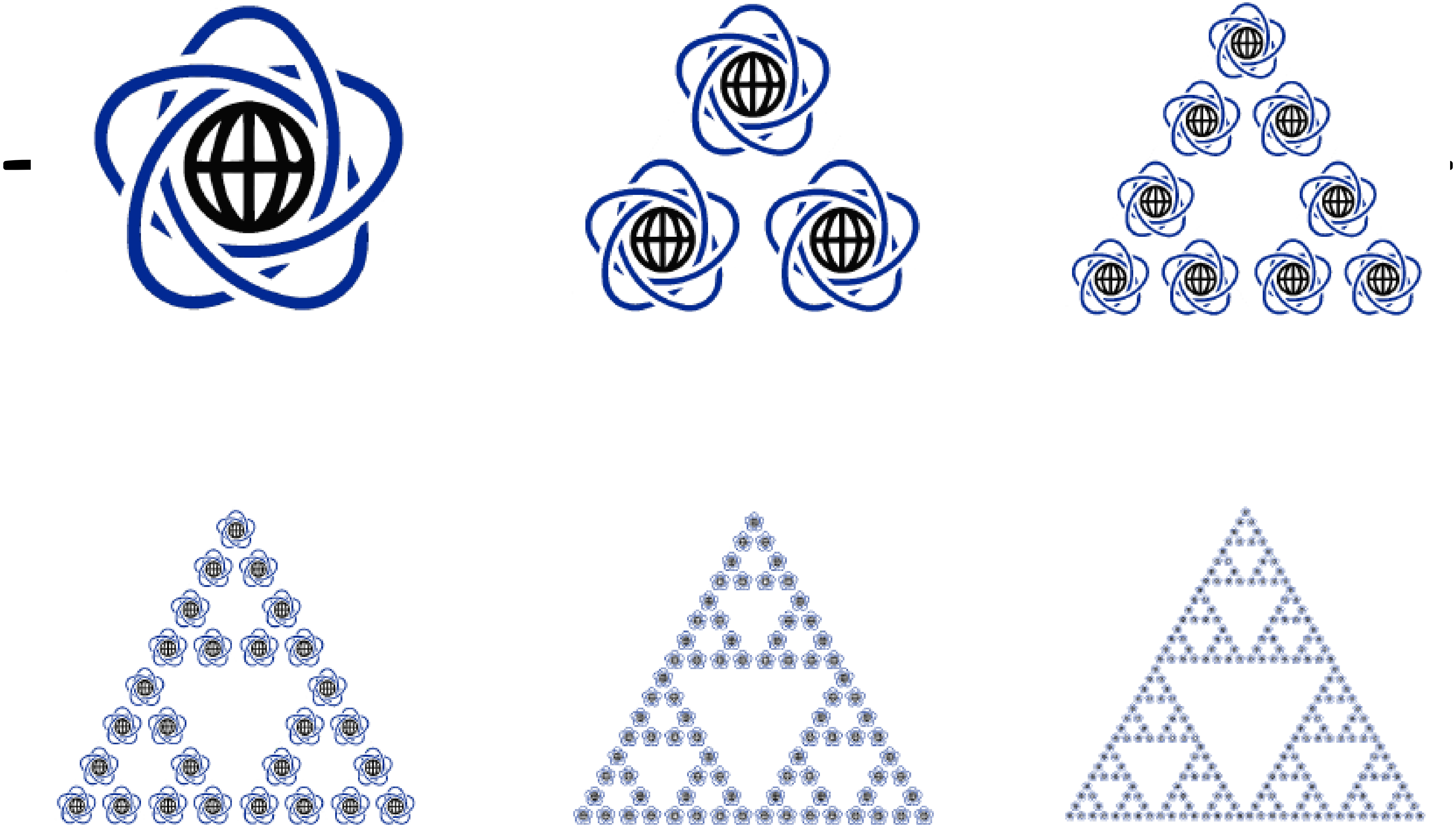
- Let f_i be contractions on (X, d). Let S be compact subset of X. Let

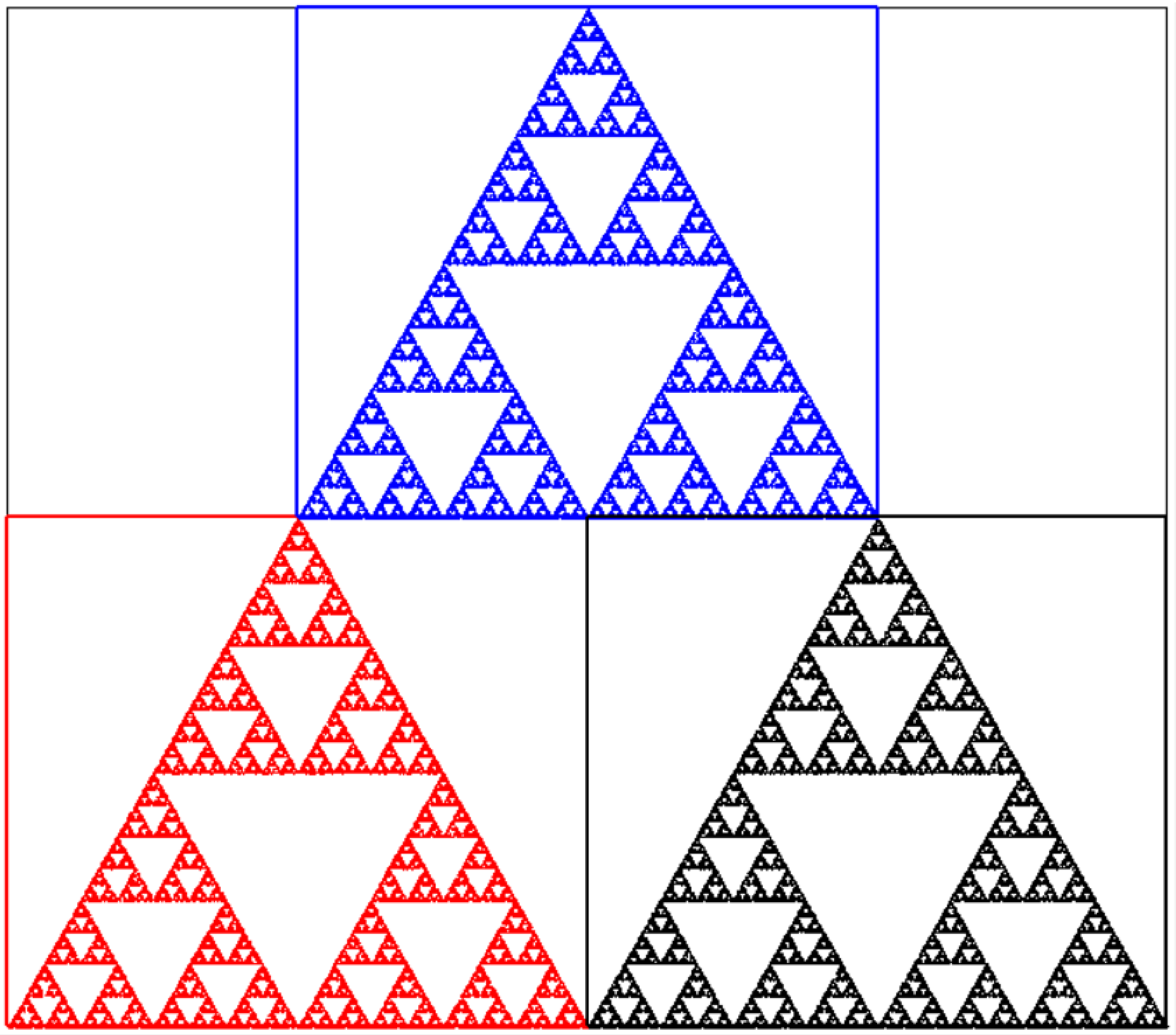
$$H(S) = \bigcup_{i=1}^{n} f_i(S).$$

Then, H is a Hutchinson operator, and it is a contraction.









- Therefore, fractals (some of them) can be viewed as fixed points of some contraction in some strange metric space.

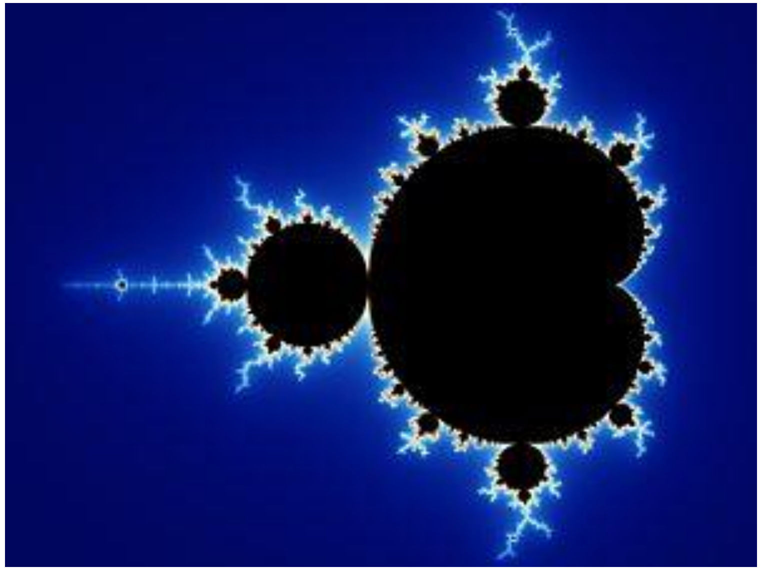
BARNSLEY FERN

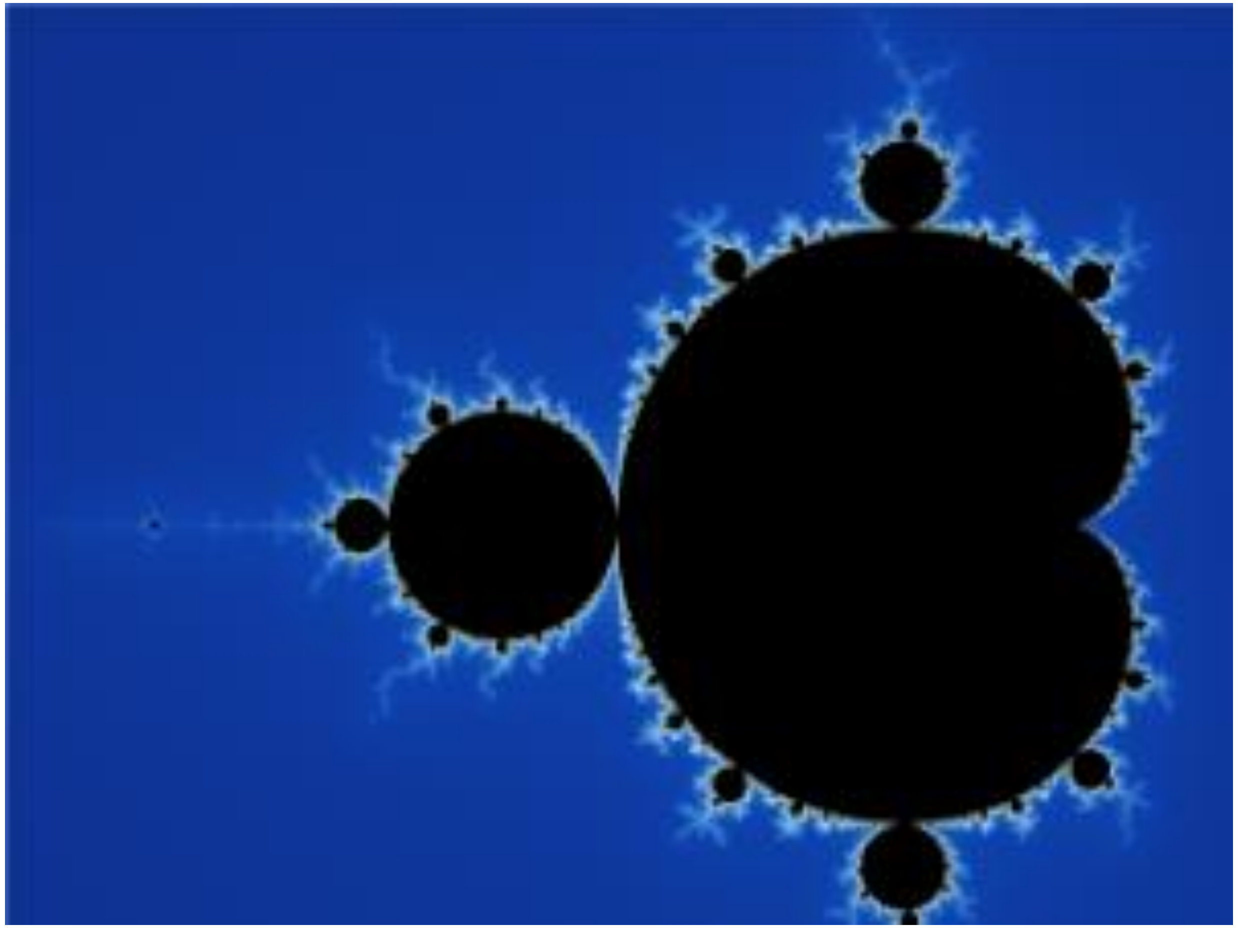
MANDELBROT SET

- This is a set of such $p \in \mathbb{C}$ that the sequence

$$z_0 = 0$$
$$z_{n+1} = z_n^2 + p$$

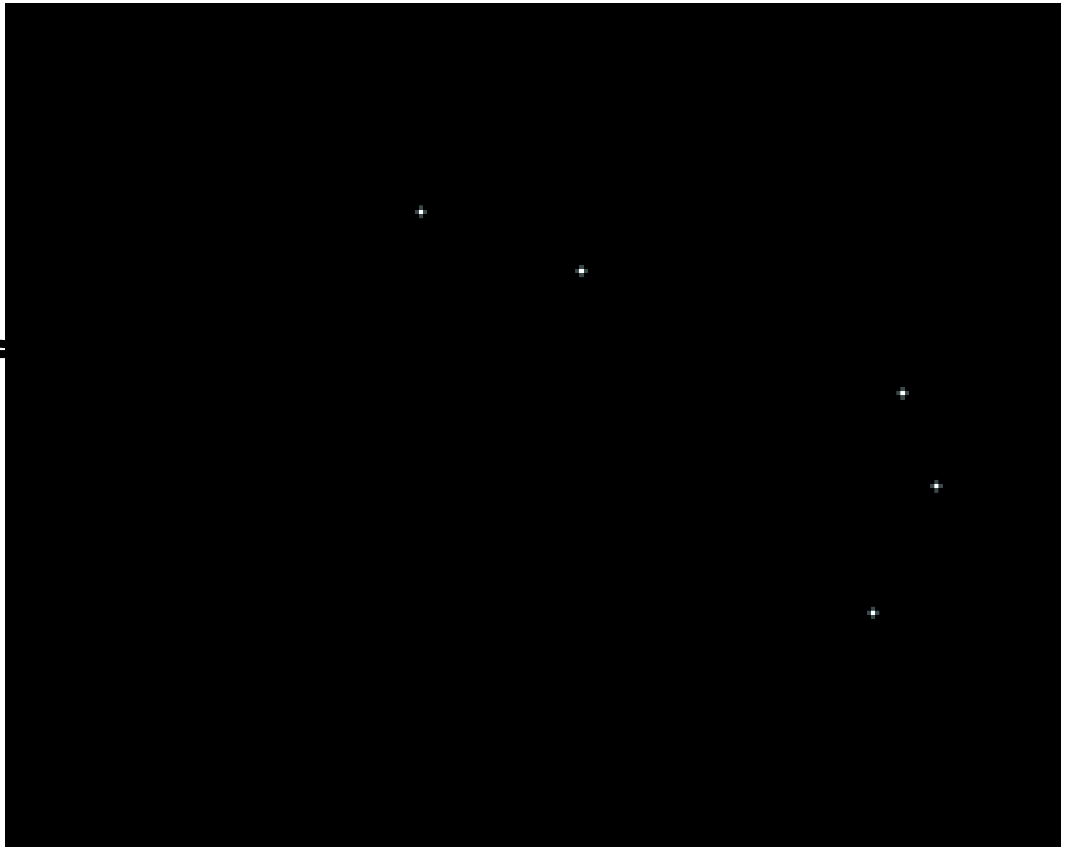
does not diverge to ∞.





IFS - ITERATED FUNCTION SYSTEM

- CHAOS GAME - pick a random point and iterate it randomly using one of basic contractions.



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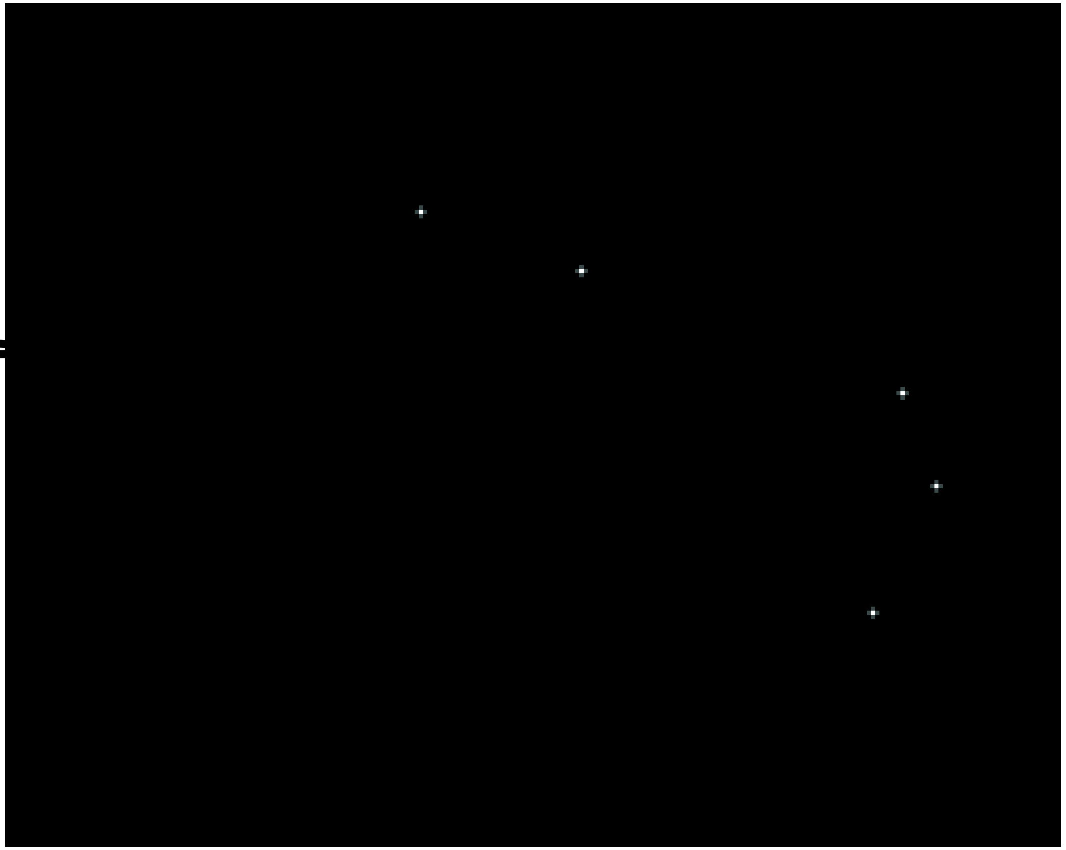

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- Very fast => FRACTAL COMPRESSION!

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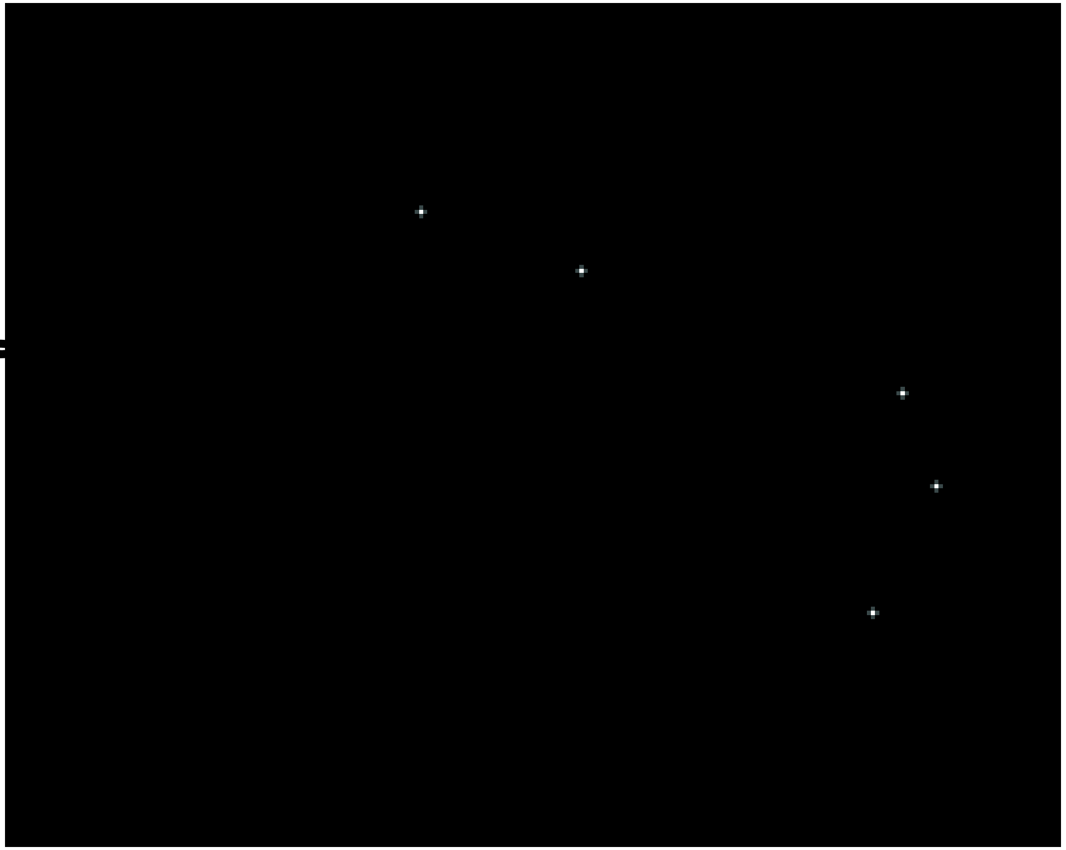

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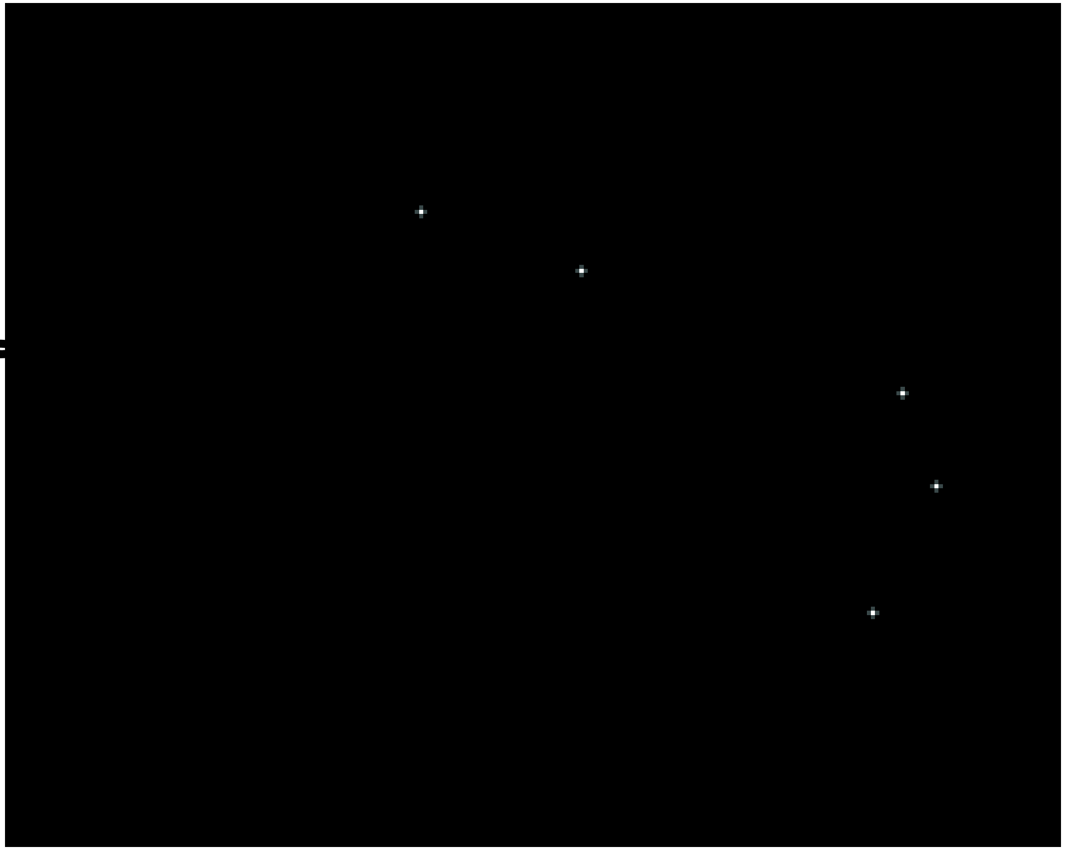

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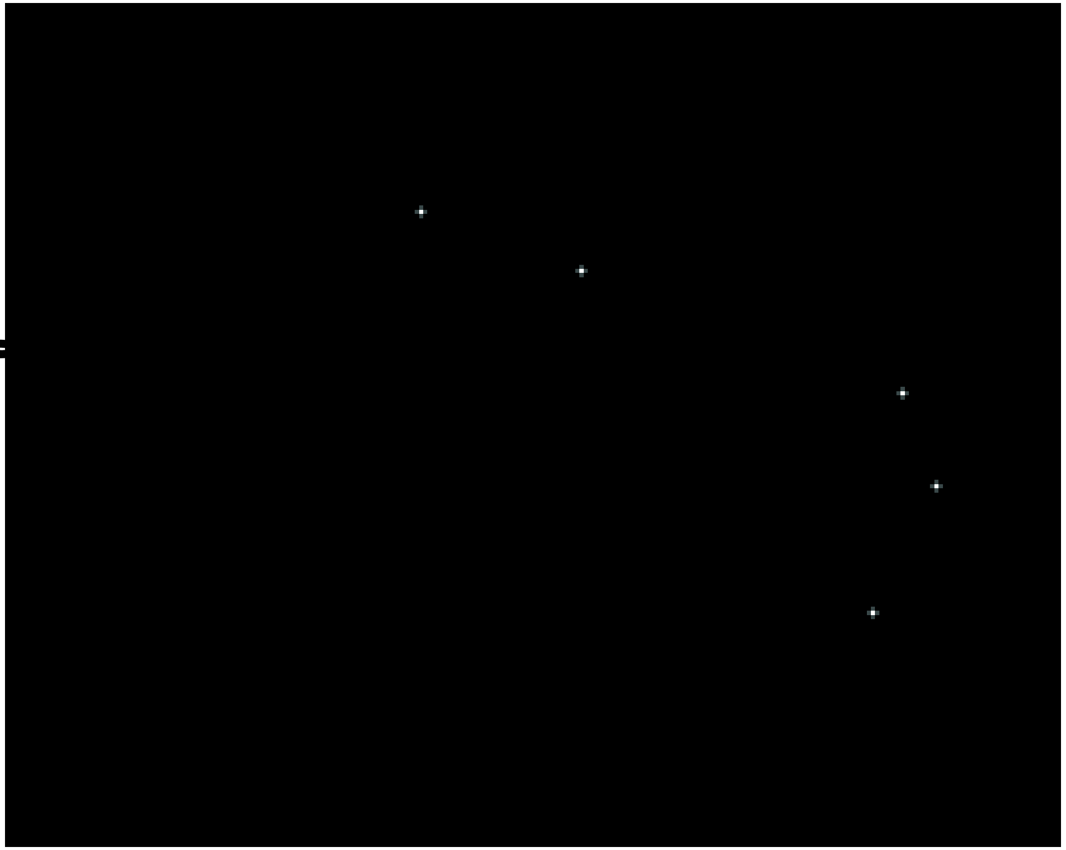

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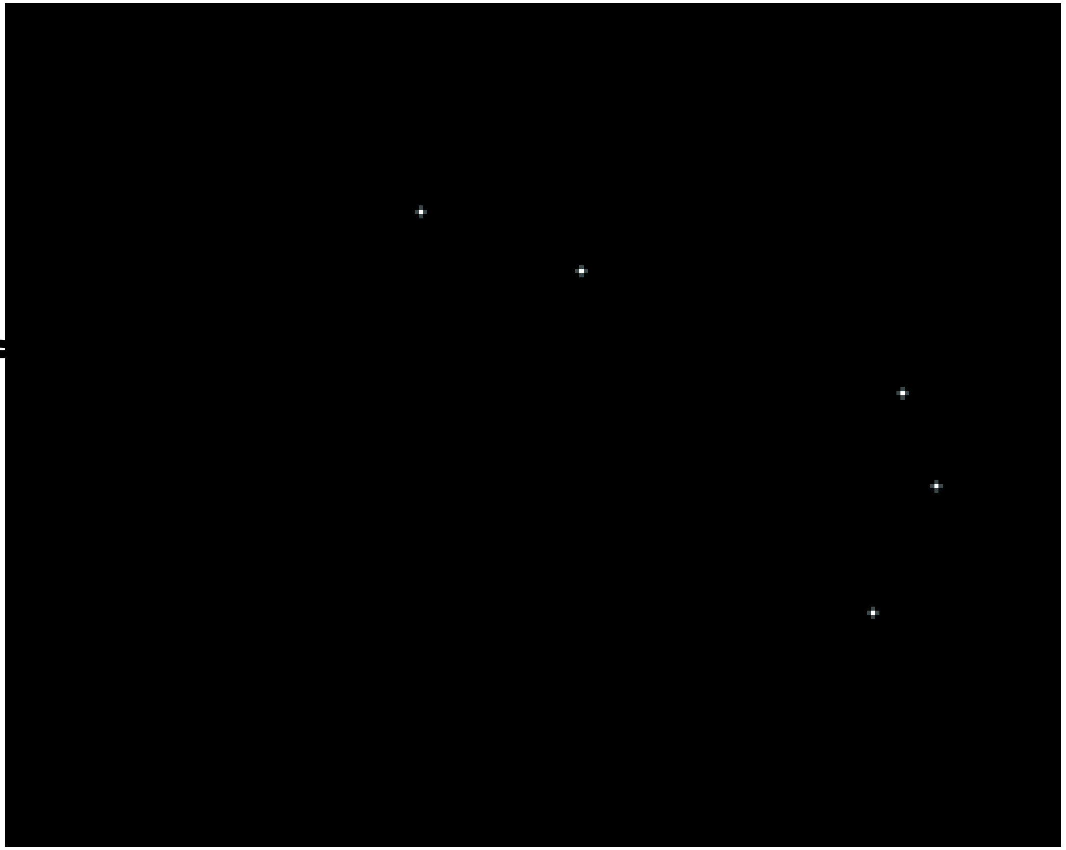

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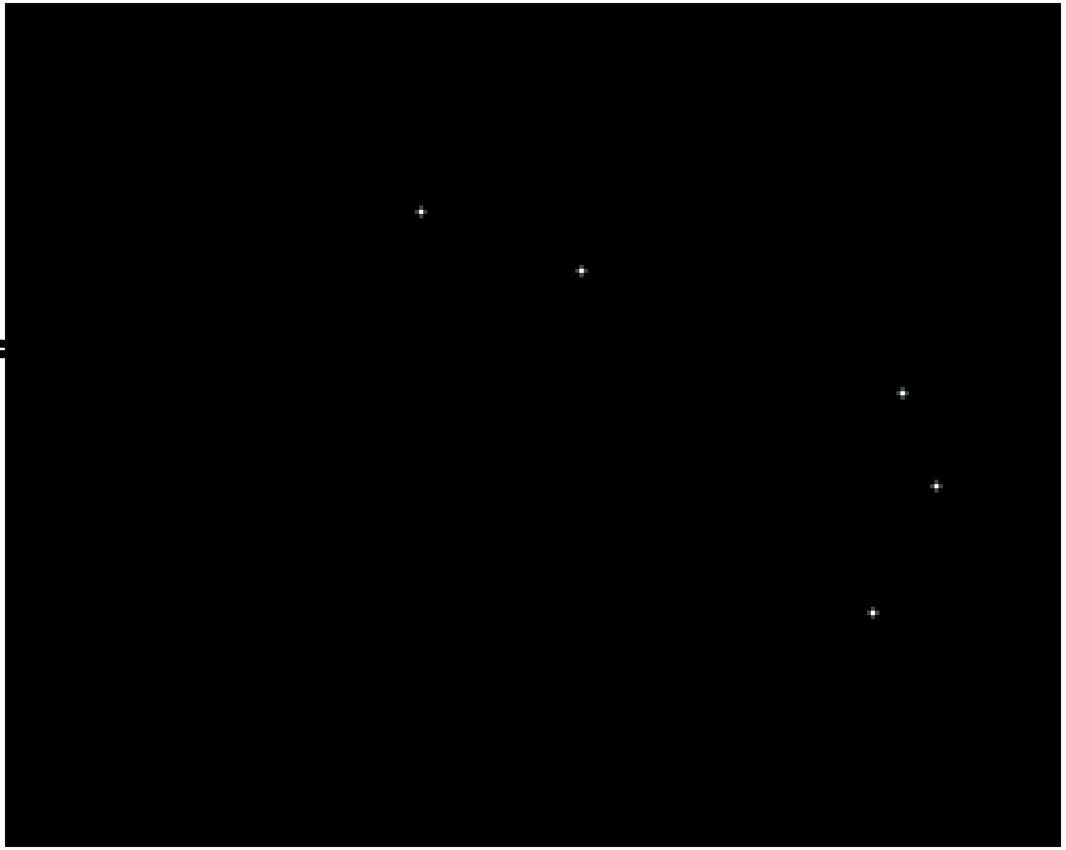

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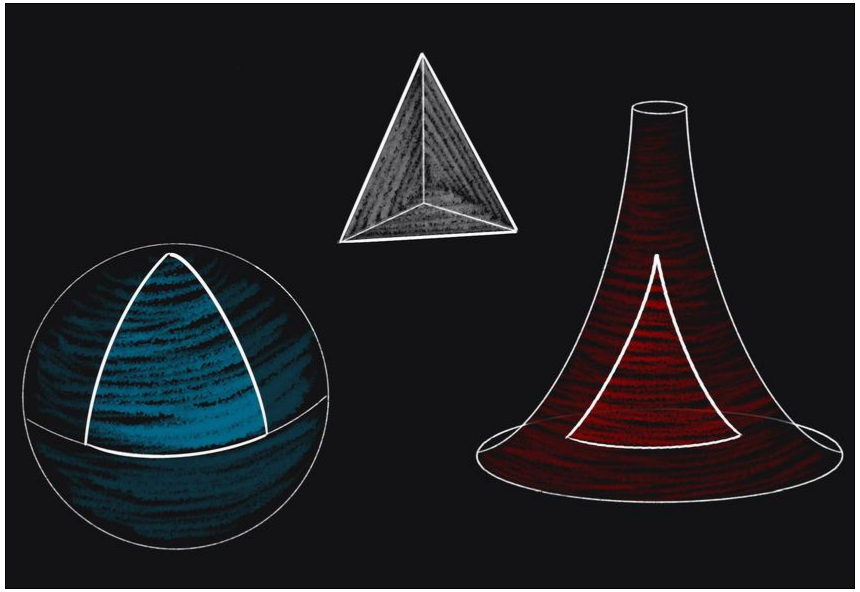
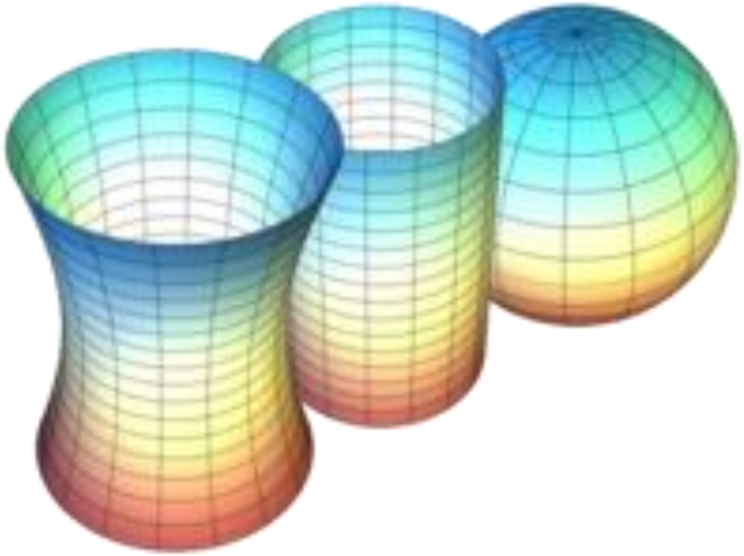
- Still an open problem.

DIFFERENTIAL GEOMETRY

- Aim: study invariants of curves, surfaces (and generalizations – manifolds) using methods involving „calculus"
- Using a definition of curvature we can examine how much a manifold deviates from being flat
- We can detect it even if we live inside such a manifold



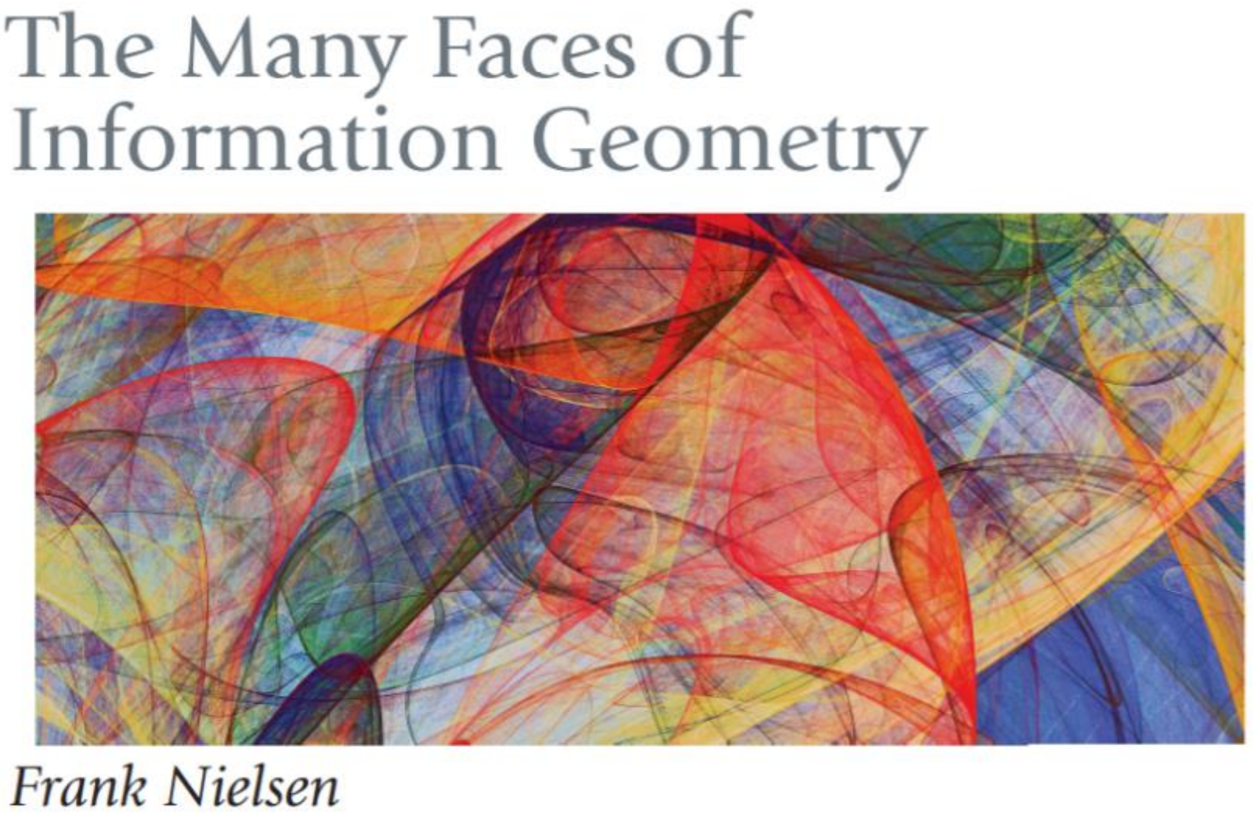


DIFFERENTIAL GEOMETRY

- General Relativity (Physics): Used to describe spacetime curvature and gravitational fields.
- Robotics: Used for motion planning, particularly in navigating curved spaces (manifolds) for robot movements.
- Computer Vision: Applied in image processing and shape recognition, where surfaces and curves are analyzed.
- Machine Learning: Used in manifold learning to explore data structures that lie on lower-dimensional surfaces.
- Economics: Employed in optimal transport theory and studying geometrical structures in decision spaces.
- Engineering: Applied in structural analysis and optimization, particularly in the study of mechanical stresses and strains.
- Biology: Used to understand the shapes of biological structures (e.g., proteins) and their transformations.
- Fluid Dynamics: Helps describe the geometry of flow and curvature in space for complex fluid behaviors.
- …

INFORMATION GEOMETRY

- Information geometry is an interdisciplinary field that applies the techniques of differential geometry to study probability theory and statistics. It studies statistical manifolds, which are Riemannian manifolds whose points correspond to probability distributions.
- Statistical inference
- Time series
- Quantum systems
- Neural networks and machine learning
- …



GPT4o:

ALGEBRAIC GEOMETRY

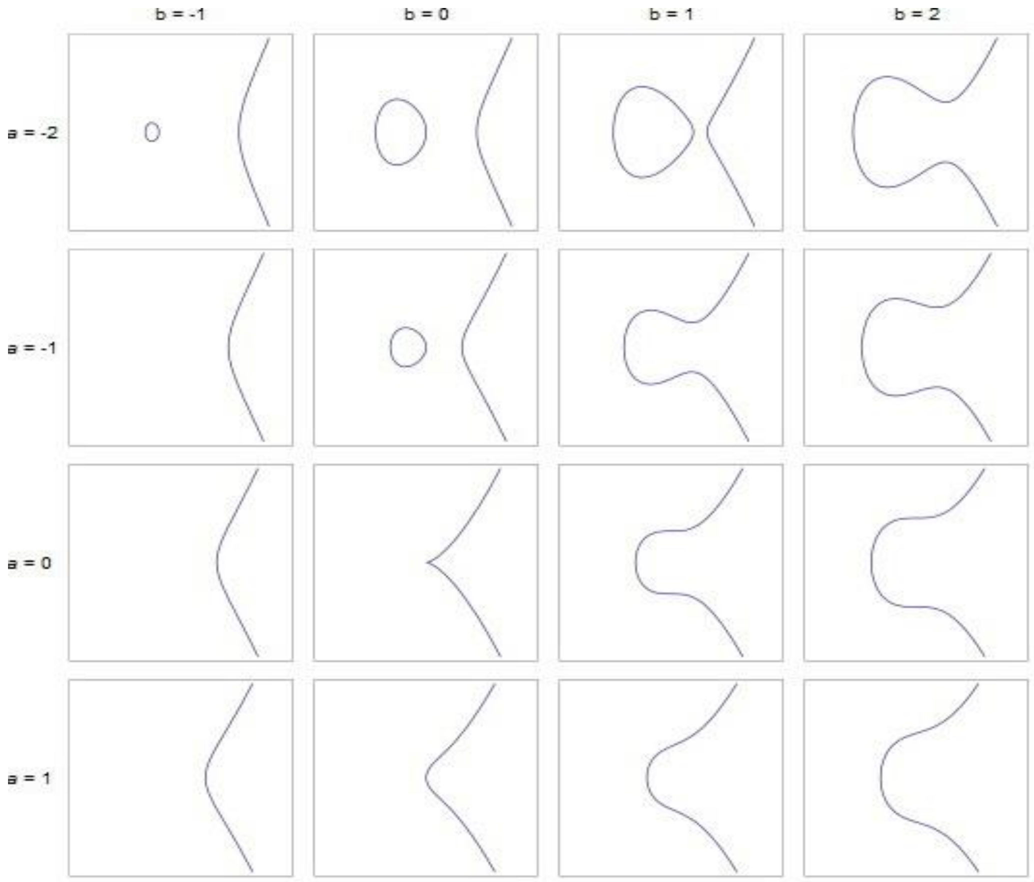
- **Varieties**: Study of solutions to systems of polynomial equations, called algebraic varieties.
- **Singularities**: Analyzing points where varieties fail to be smooth, and methods to resolve these singularities.
- **Arithmetic Geometry**: Combining algebraic geometry with number theory to study solutions of polynomial equations over different fields (like integers).

ALGEBRAIC GEOMETRY

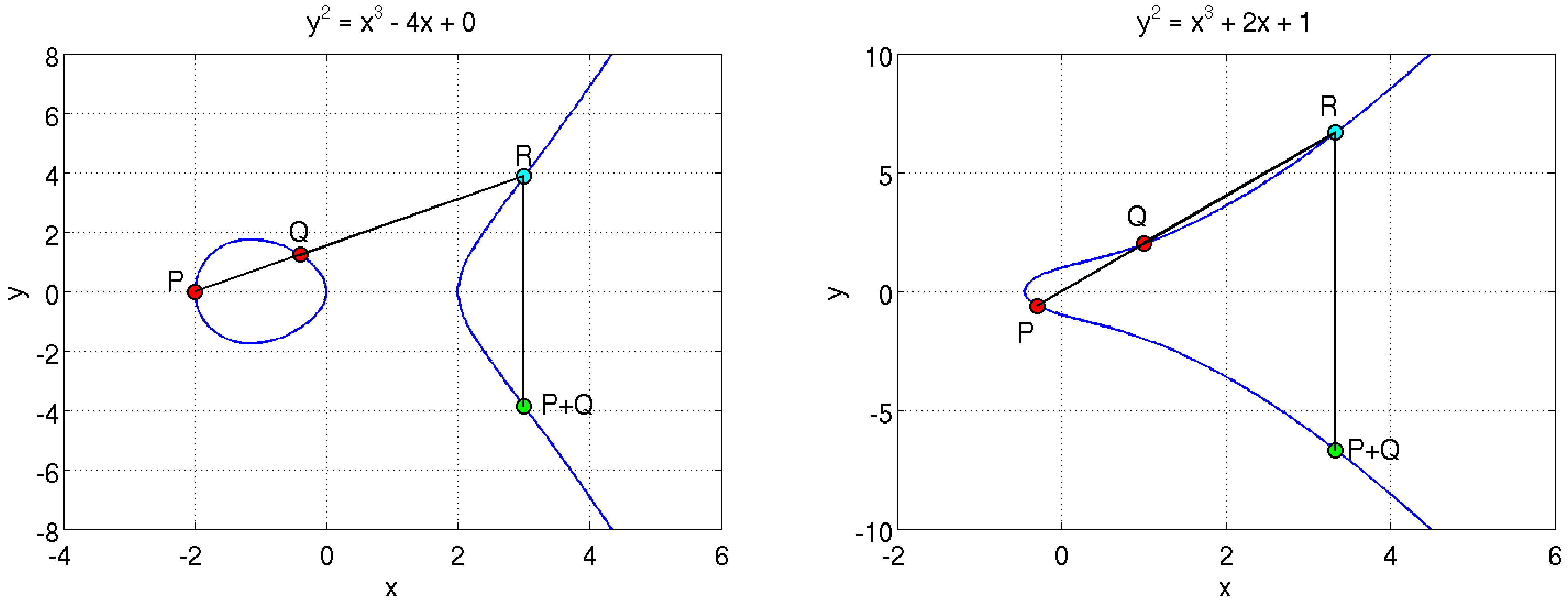
- **Cryptography**: Used in elliptic curve cryptography, where algebraic curves provide secure methods for encrypting data.
- **Coding Theory**: Applied in constructing error-correcting codes using algebraic curves, improving data transmission accuracy.
- **Robotics**: Useful in solving polynomial equations related to robot kinematics and motion planning.
- **Machine Learning**: Applied in the study of data manifolds and in defining features using algebraic invariants.
- **Biology (Phylogenetics)**: Used to model evolutionary trees through algebraic varieties representing genetic relationships.
- **Chemistry (Crystallography)**: Helps in understanding molecular structures and symmetries through the study of algebraic varieties.
- ….

ELLIPTIC CURVE CRYPTOGRAPHY

- Curves of the form $y^2 = x^3 + ax + b$.
- Forms an „abelian group"
- Symmetric about the x-axis.
- Point at Infinity acting as the identity element.

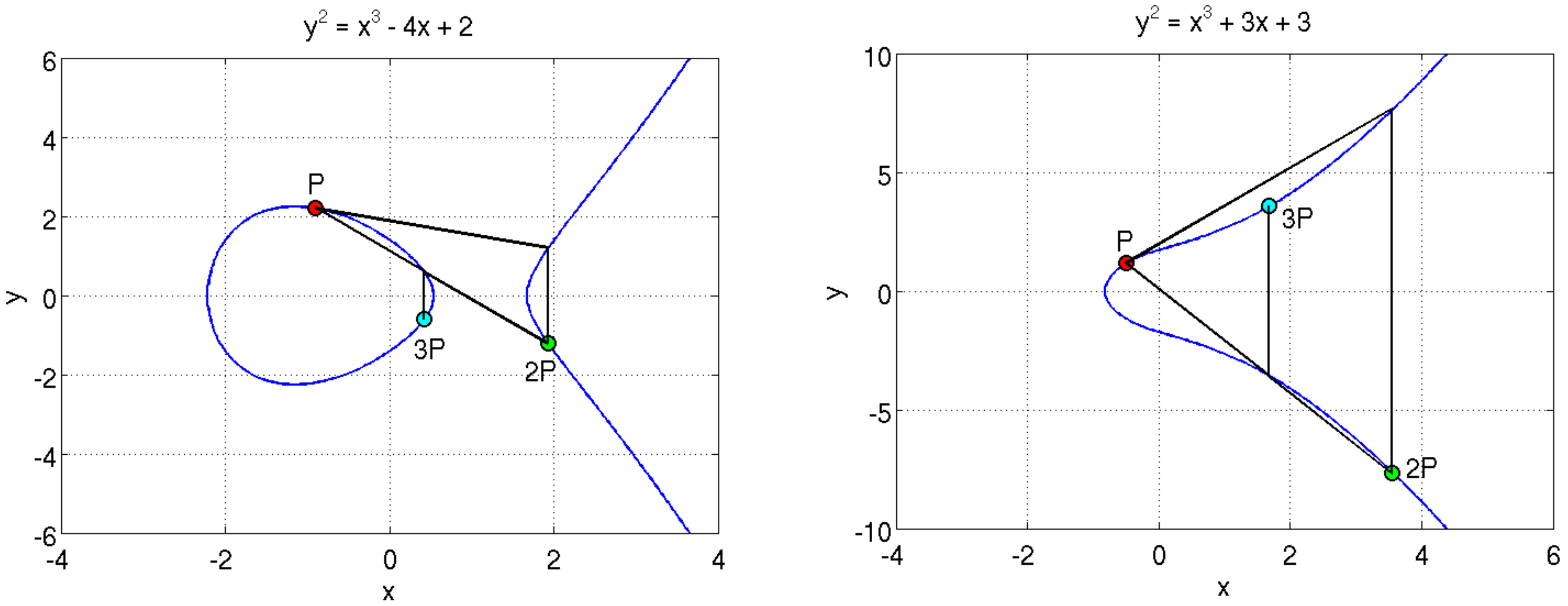


HOW TO ADD?



- P and Q added to obtain P+Q which is a reflection of R along the x-axis

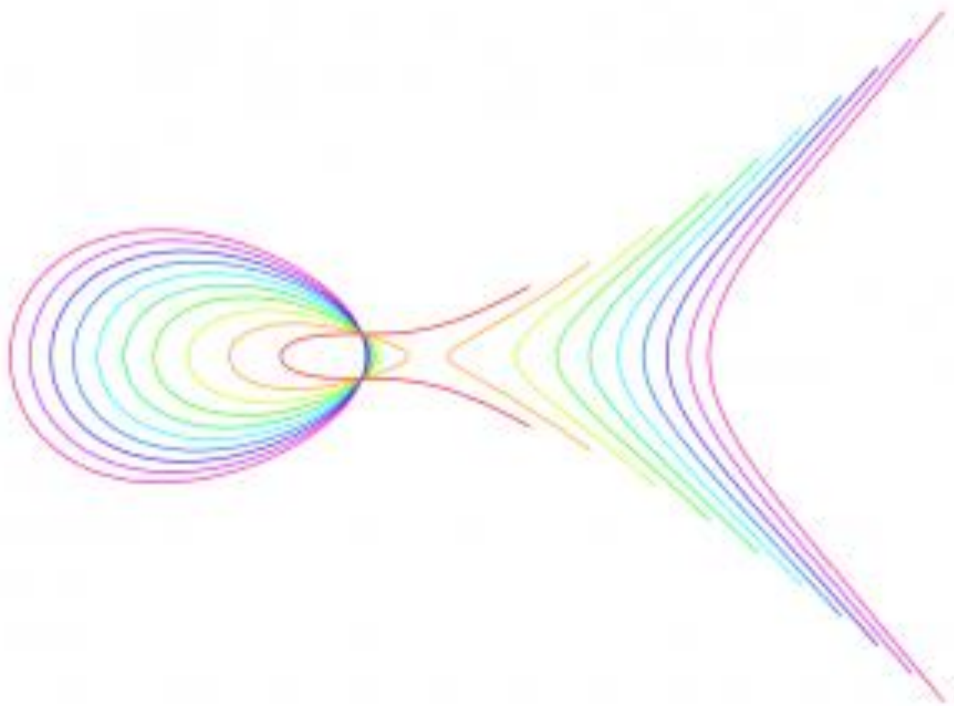
HOW TO ADD?



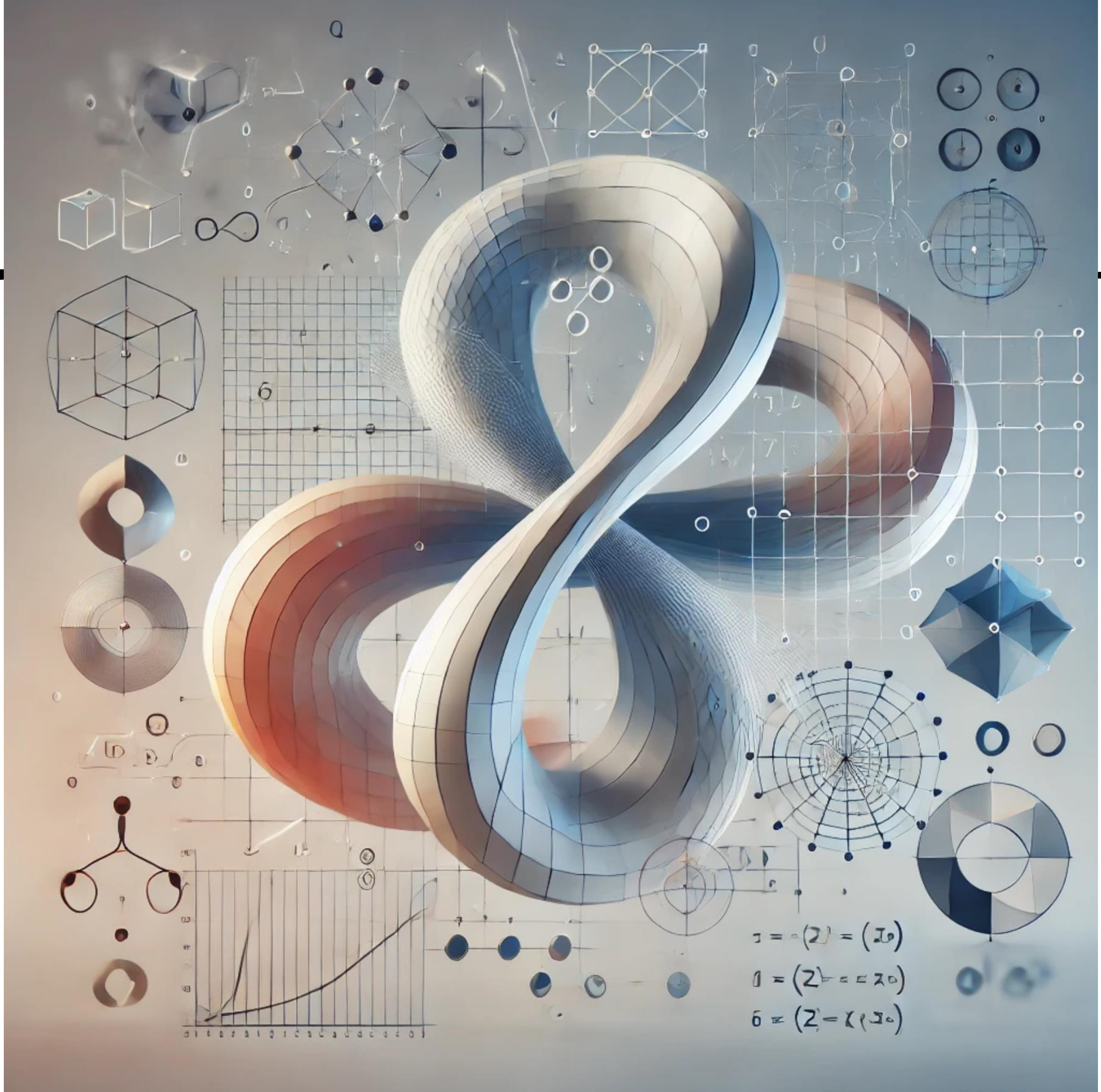
- If we want add P+P we start from drawing a tangent line at P.

DISCRETE LOG PROBLEM

- If Q=kP and we are given Q and P, it is hard to find k.
- Methods to solve include brute force and some other ways, but up to this moment, they are computationally expensive or unfeasible
- Exponential running time



GPT4o:

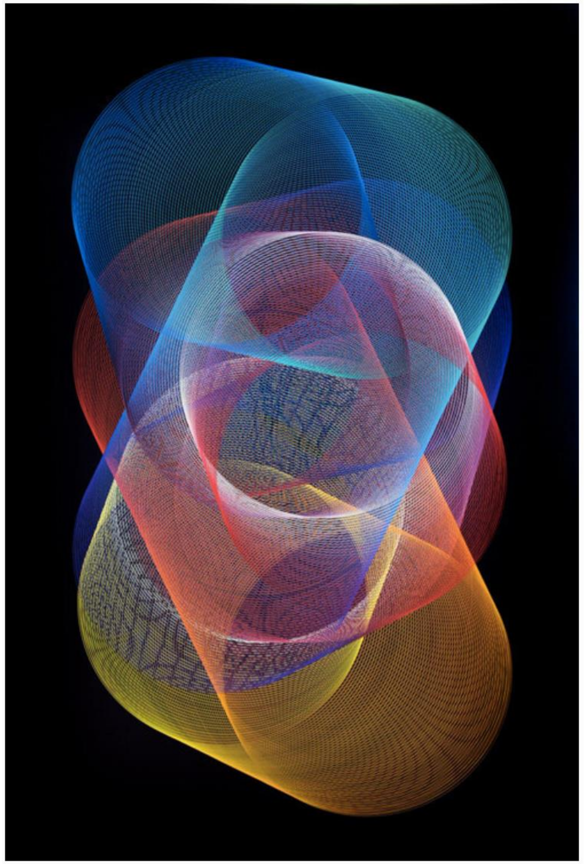


STEAM

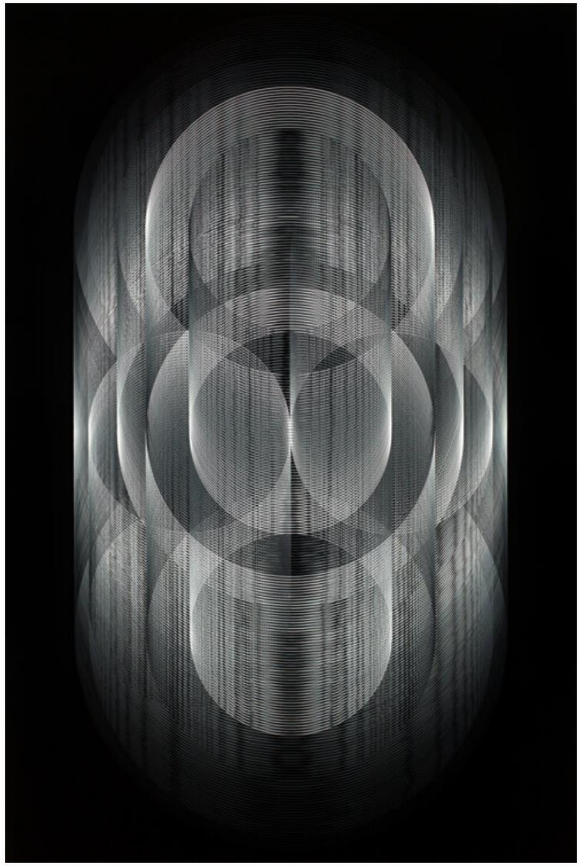
An educational approach to learning that uses **Science, Technology, Engineering, the Arts and Mathematics** as access points for **guiding student inquiry, dialogue, and critical thinking.**

ART, SEBASTIEN PRESCHOUX

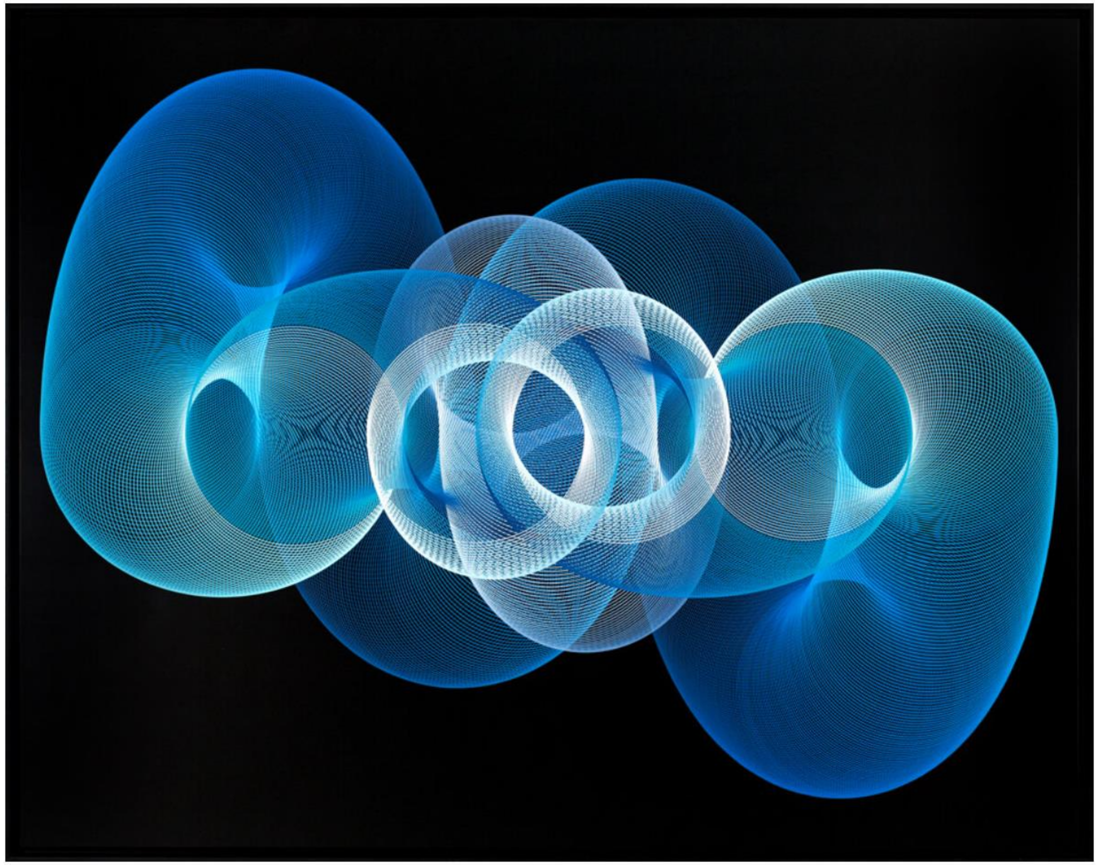
- https://www.sebastienpreschoux.com/#/paintings/



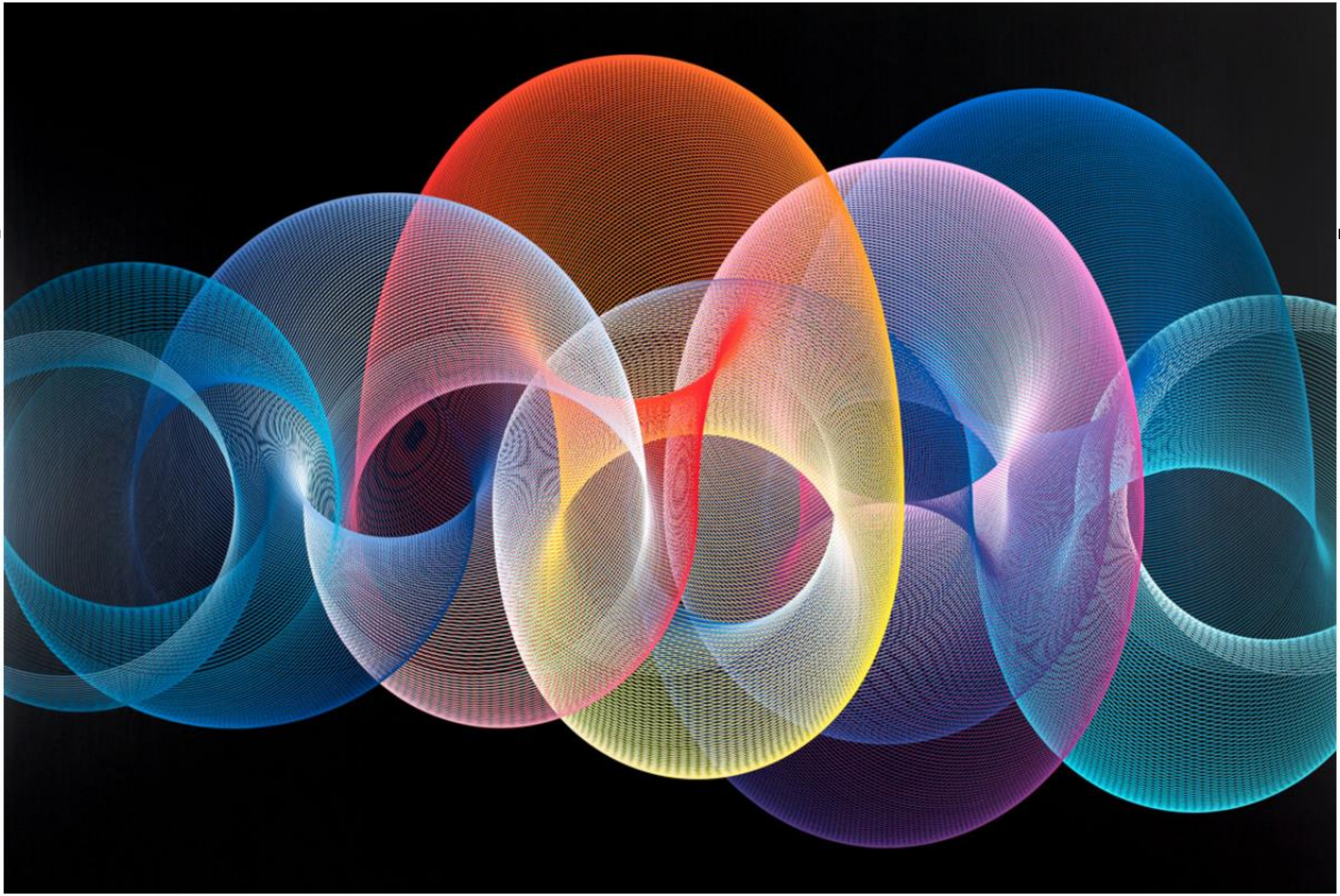
AVALON



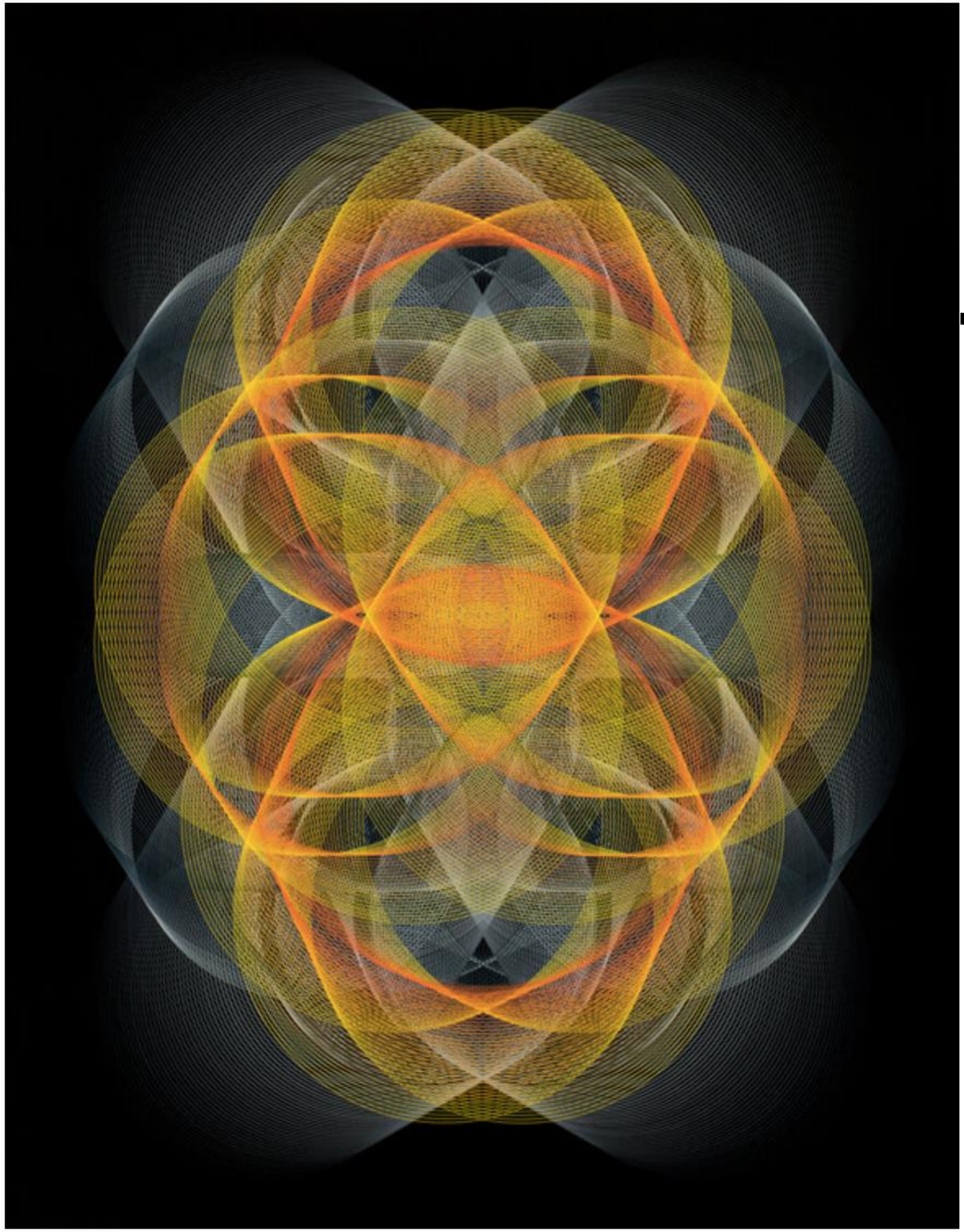
CHROMO-VIBRATILE



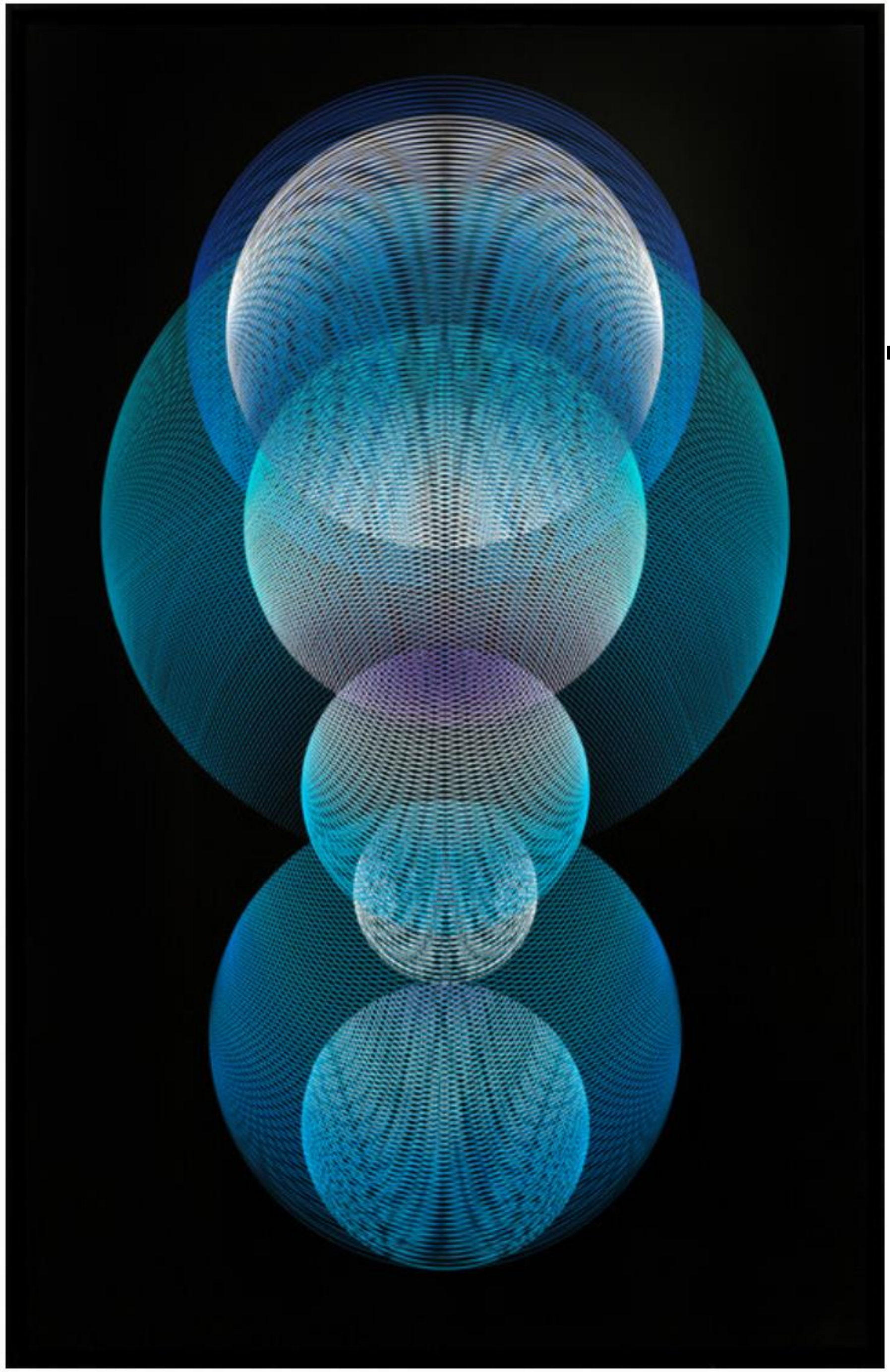
ACONITE



KHRÔMA_18



RISING

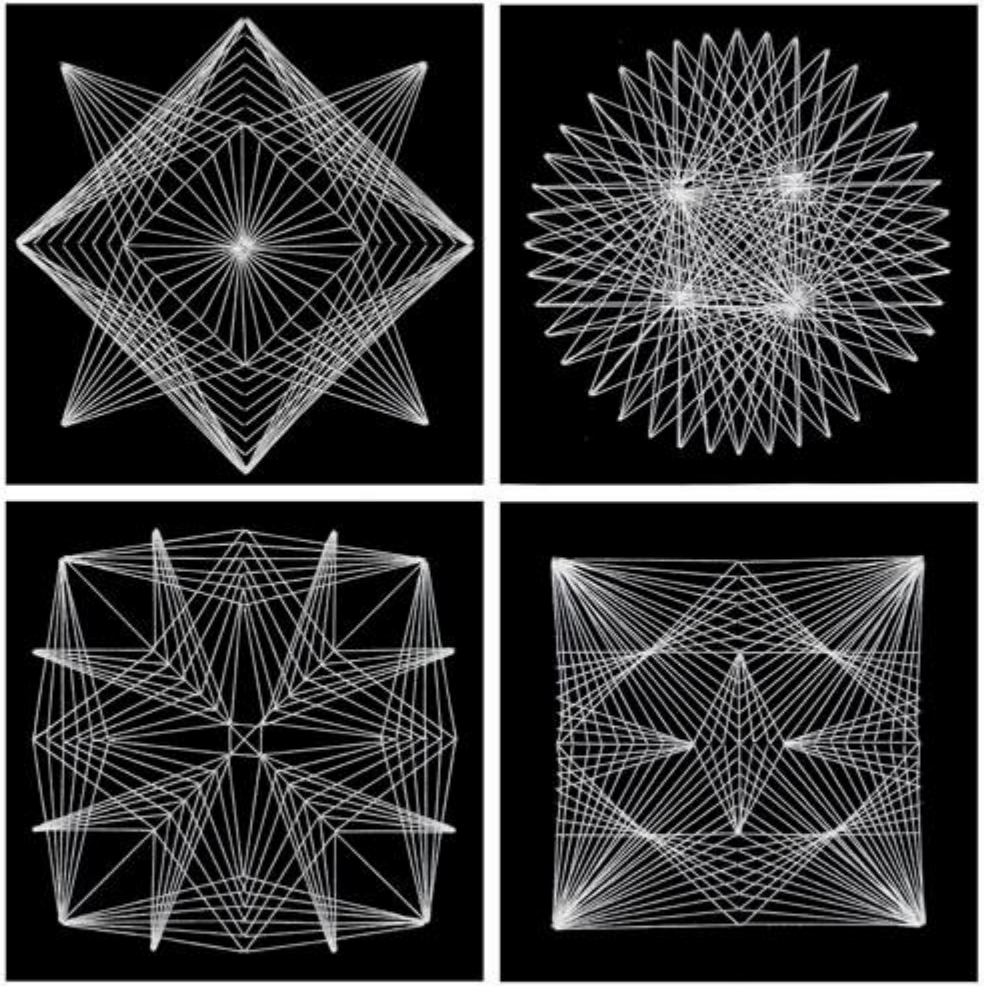


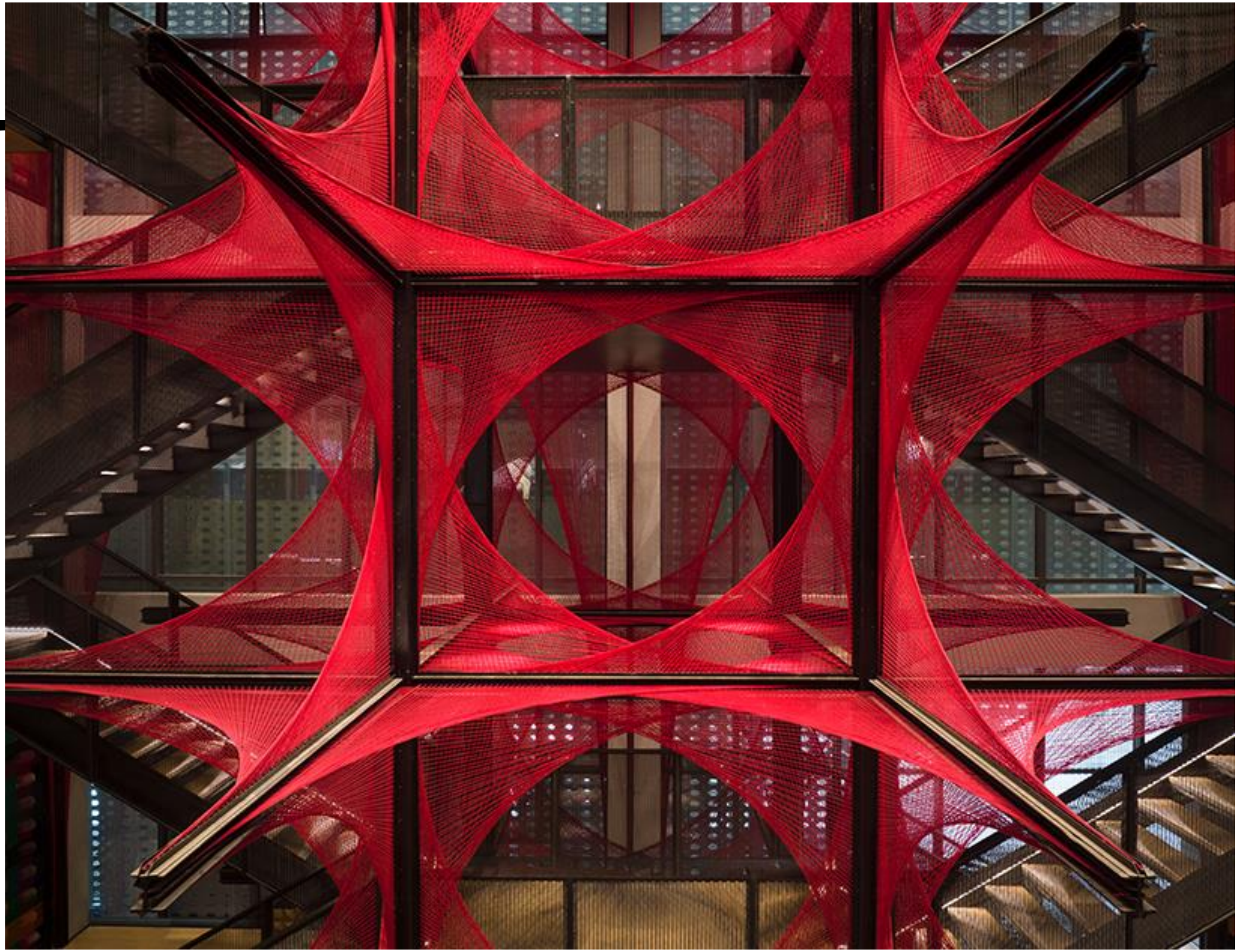
VARIATION GRAFIK n°5

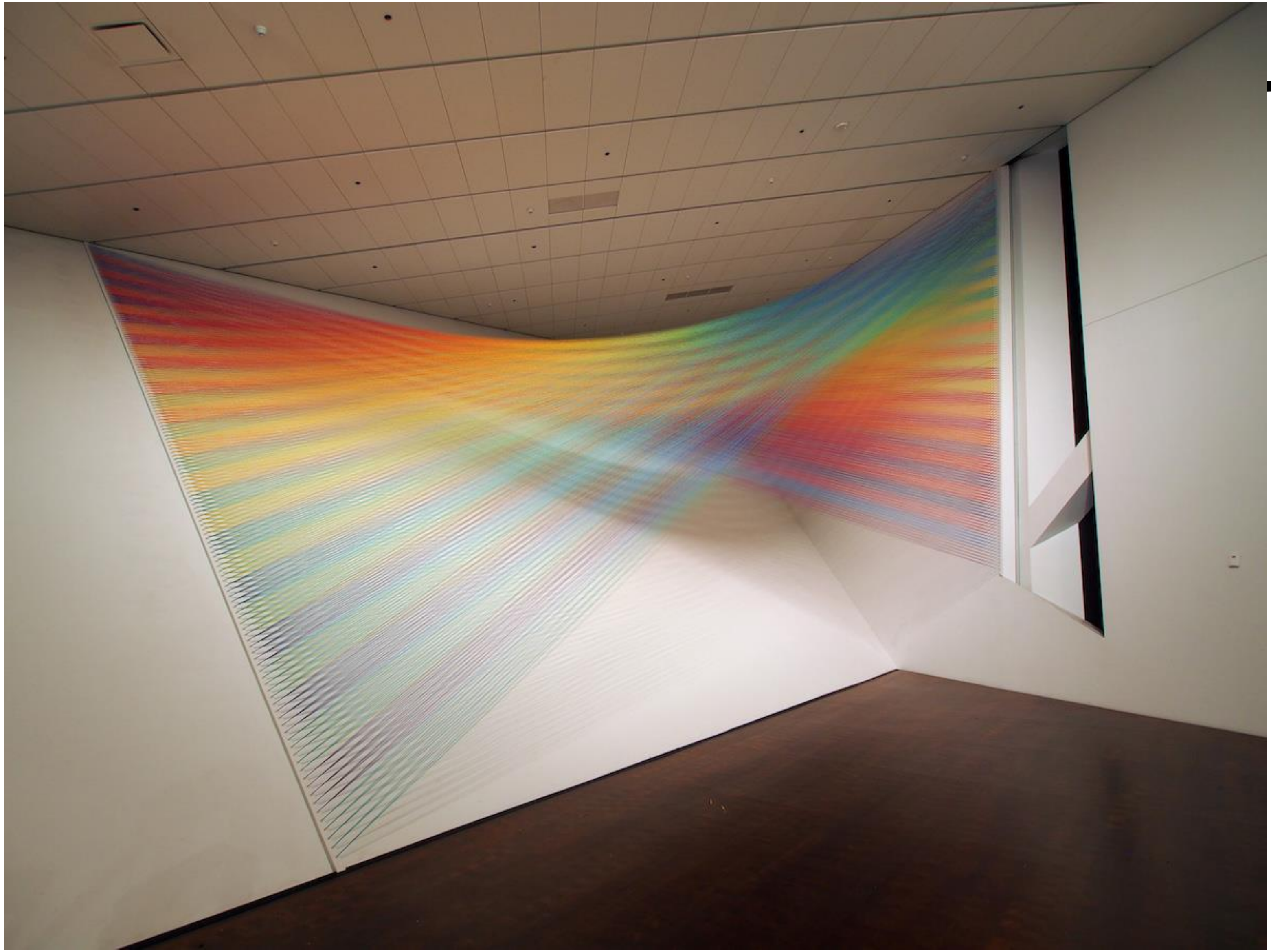
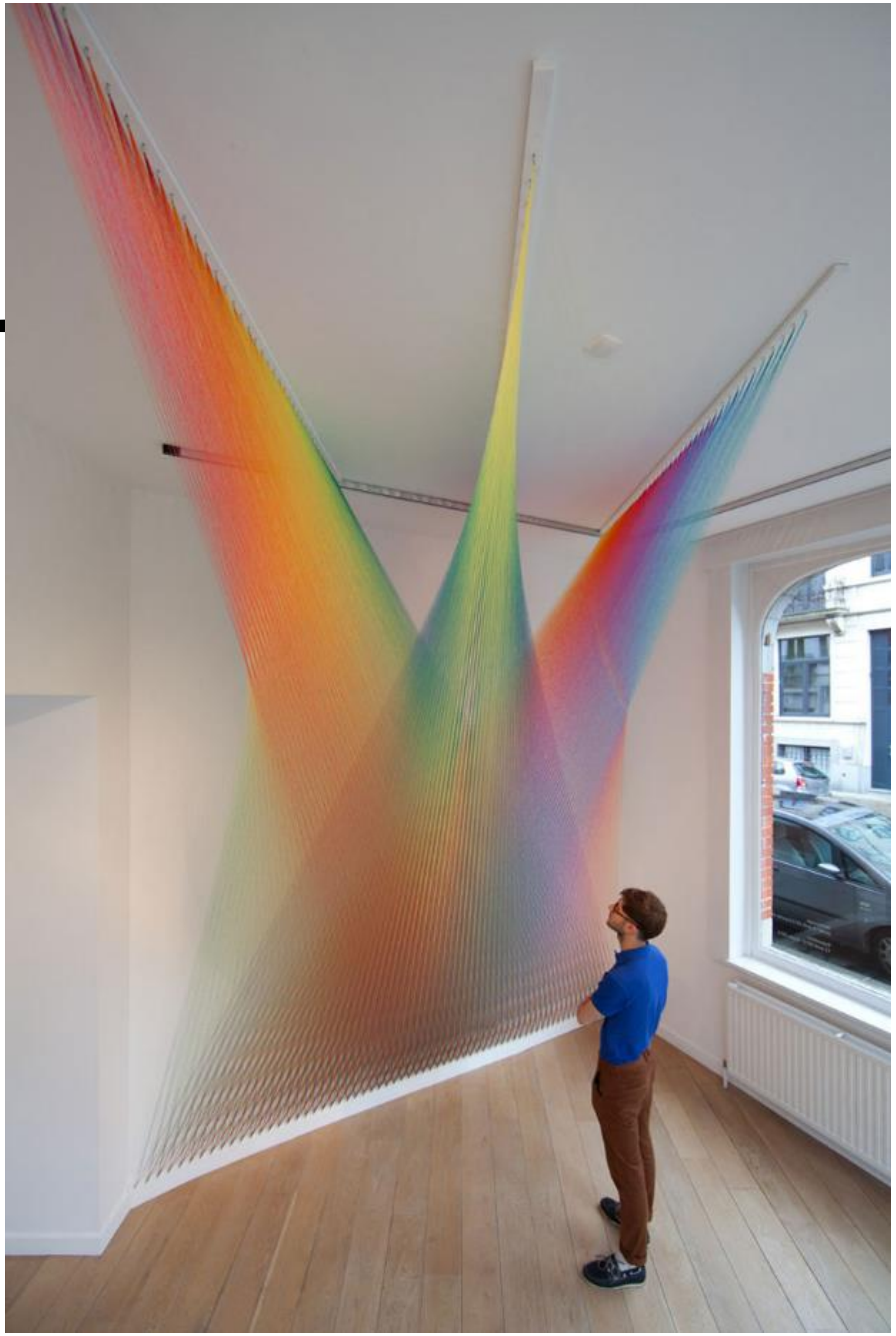
BRIDGE paris

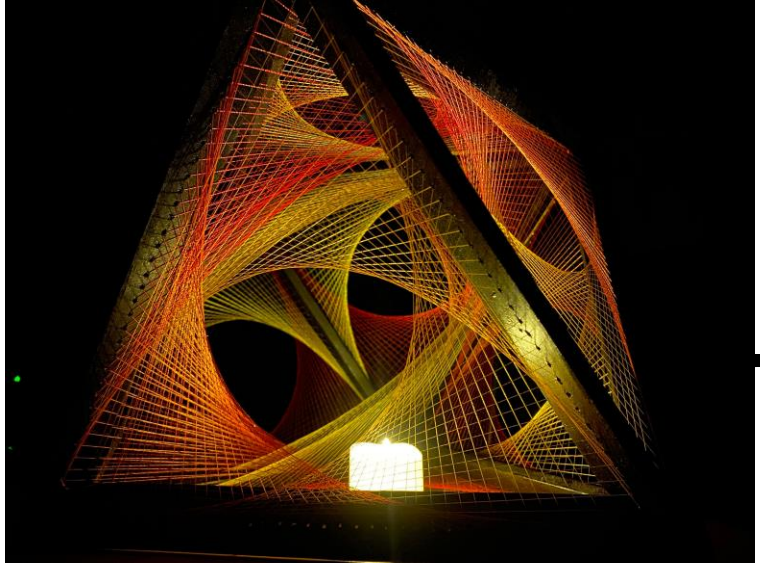
WATER LILY

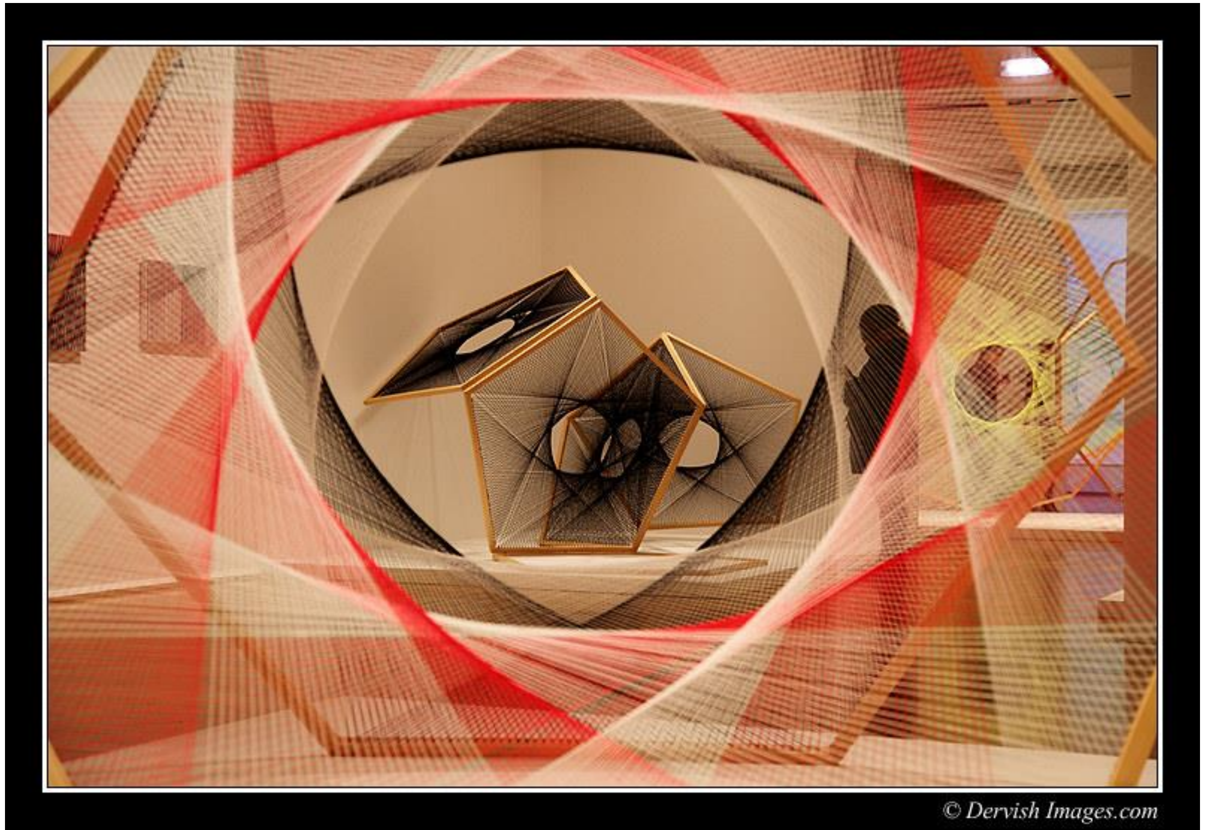
MORE ON STRING ART





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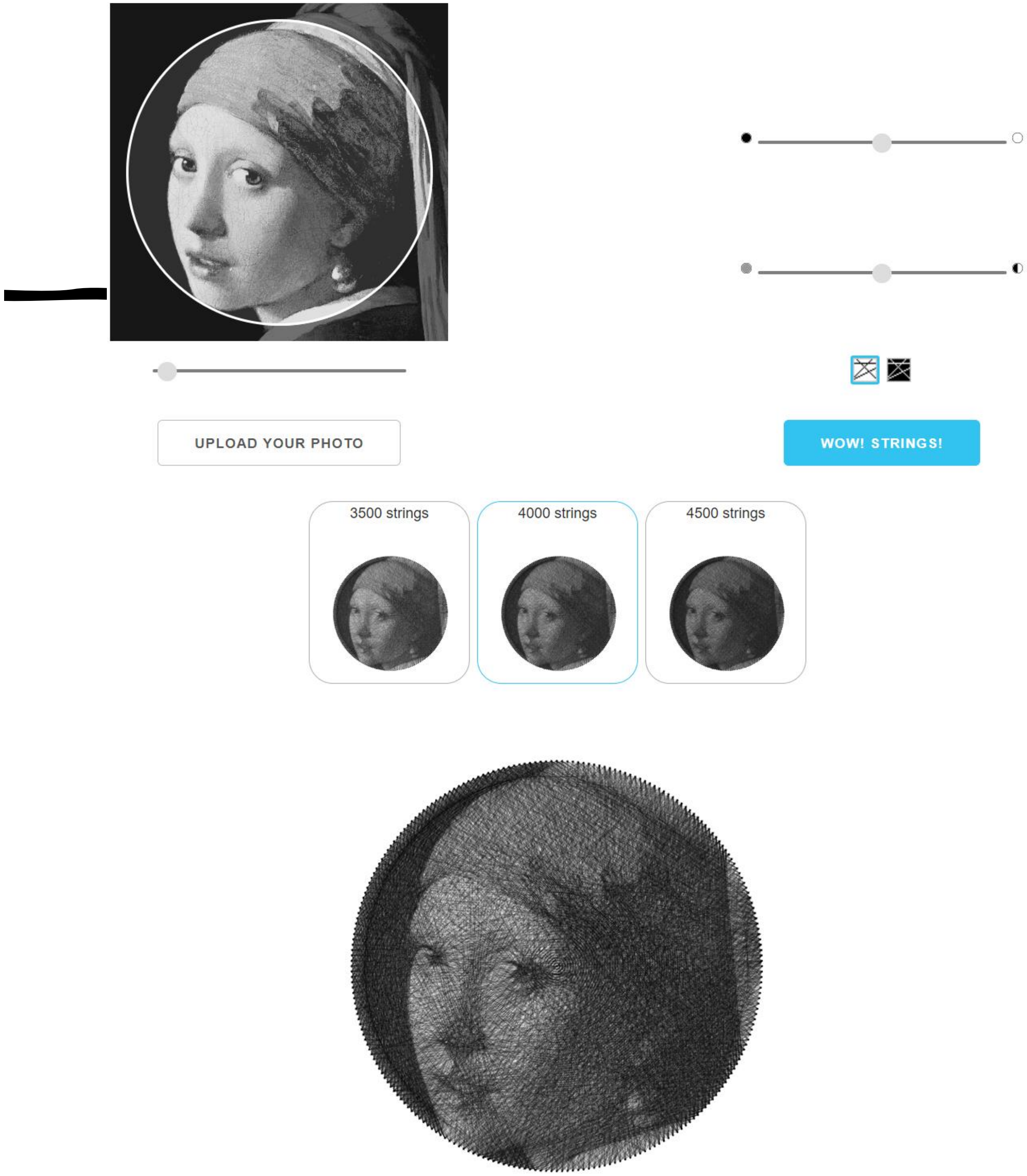
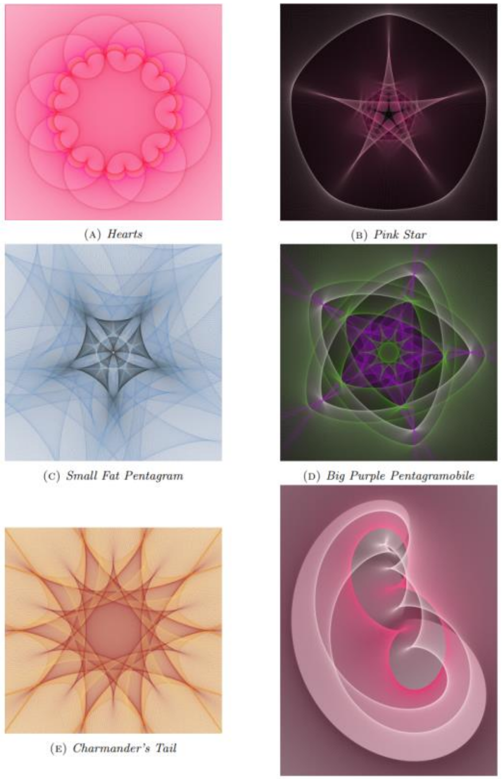
STRING ART GENERATOR

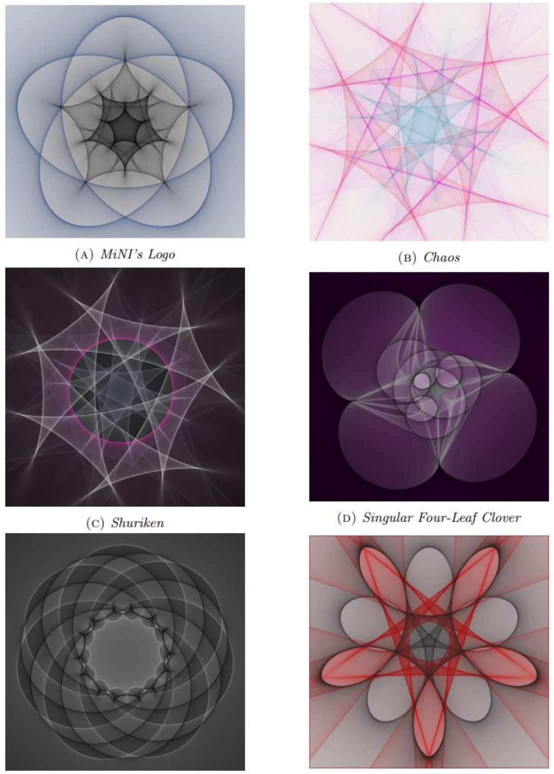


MY OWN SMALL CONTRIBUTION

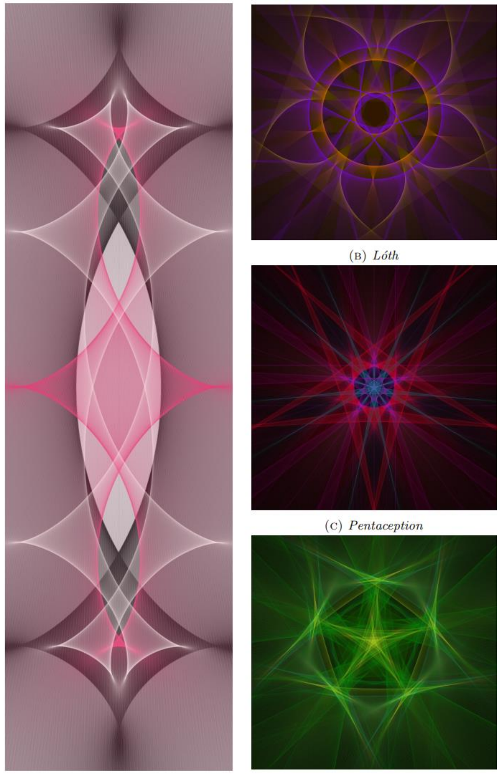
- I. Danielewska, D. Polawski, D. Sterczewska, M. Zwierzynski: "Arthistic Aspects of the Wigner Caustic and the Centre Symmetry Set", arXiv:2409.04443, sent to Journal of Mathematics and the Arts (hope they will like it)



(A) *Hearts* (B) *Pink Star* (C) *Small Fat Pentagram* (D) *Big Purple Pentagramobile* (E) *Charmander's Tail* (F) *The Beginning*



(A) *MiNI's Logo* (B) *Chaos* (C) *Shuriken* (D) *Singular Four-Leaf Clover* (E) *Black and White* (F) *Beast's Rosette*



(A) *Harmony* (B) *Lóth* (C) *Pentaception* (D) *Pestilence*

WHY DO WE NEED MODERN GEOMETRY?

Michal Zwierzynski

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Polish-Japanese Singularity Working Days 2024

09-14 September 2024